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OF THE

ROYAL AERONAUTICAL SOCIETY

(FOUNDED 1897 in succession to the ANNUAL REPORTS)

Edited for the Council by J. LAURENCE PRITCHARD, Honorary Fellow

No. 158 FEBRUARY, 1924 VOL. XXVIII

NOTICES

Colonel J. L. B. Templer

Members will learn with great regret of the death of Colonel J. L. B. Templer, who will long be remembered for his work as Superintendent of the Balloon Factory at Farnborough. Colonel Templer was for many years closely associated with the Society, occupying a seat on the Council for nearly 30 years.

Election of Members

The following Members were elected at a Council meeting held on Tuesday, January 8th :—

Fellow.—J. D. Siddeley, C.B.E.
Associate Fellow.—F. J. W. W. Digby.
Member.—T. A. Gladstone.
Associate Member.—A. G. Lamplugh.
Foreign Member.—Major N. Brearley.

International Conference on Applied Mechanics

An International Conference on Applied Mechanics will be held at Delft (Holland) from April 22nd to April 26th this year. Members wishing to participate should communicate with Prof. Dr. J. M. Burgers, 76, Nieuwe Laan, Delft. The English representatives on the committee are Professor L. Bairstow, Professor E. G. Coker, Dr. A. A. Griffith, Major R. V. Southwell and Dr. G. I. Taylor.

Lantern Slides

A number of slides of light aeroplanes has been presented to the Society's loan collection by the Chairman.

Journal Binding

Members are reminded that arrangements have been made for the binding of complete annual volumes of the Journal in blue cloth cases with gilt lettering at a charge of 4s. 6d. per volume, including the supply of the case. Members who desire to take advantage of this arrangement should forward their sets direct to The Lewes Press, Ltd., High Street, Lewes, at the same time sending a remittance for 4s. 6d. to the Secretary at the Society's offices. A note stating the name and address of the sender should be included in the parcel to the binders. The complete volumes will be returned direct to members postage paid.

Individual sets of earlier volumes can be bound in the same style at a price of 5s. 6d. per volume, including supply of binding case.

Annual Subscriptions

Members who have not yet paid are reminded that their subscriptions became due on January 1st, irrespective of the date of election.

Forthcoming Arrangements

Thursday, Feb. 7th, 5.30 p.m.,* Squadron Leader Maycock, "Airmanship at Sea."

Tuesday, Feb. 12th, 5.30 p.m., Council Meeting.

Tuesday, Feb. 12th, 8.0 p.m., SCOTTISH BRANCH, Mr. A. E. L. Chorlton, "The Beardmore 1,000 h.p. Engine."

Thursday, Feb. 21st, 5.30 p.m.,* Mr. H. Hamshaw Thomas, "Aerial Photography and Survey."

Thursday, March 6th, 5.30 p.m.,* Major Tucker, "Sound Reception."

Tuesday, March 11th, 8.0 p.m., SCOTTISH BRANCH, Mr. W. H. Sayers, "Gliders."

* At the Royal Society of Arts, 18, John Street, Adelphi, W.C.

Cambridge University Aeronautical Society

The following is the complete programme of lectures arranged for the Spring Session :—

23rd Jan.—Mr. Alan J. Cobham, "Aerial Touring in Europe and the Near East."

30th Jan.—His Grace the Duke of Sutherland, "Some Aspects of Aviation."

6th Feb.—Mr. R. Chadwick, A.F.R.Ae.S., "Estimating Aeroplane Performances."

13th Feb.—Commander C. D. Burney, C.M.G., R.N., M.P., "Airships in Relation to Imperial and Commercial Problems."

20th Feb.—Professor B. Melvill Jones, M.A., A.F.C., "The Control of Aeroplanes."

27th Feb.—Mr. W. Sholto Sheppard, "A New Type of Commercial Aircraft."

5th Mar.—Cinematograph Films, etc.

12th Mar.—Major J. H. Ledeboer, M.B.E., "Safety Precautions in Aeroplanes."

Date to be announced later.—Mijnheer A. H. G. Fokker, "Recent Developments in Aviation."

All meetings commence at 8.30 p.m.

W. LOCKWOOD MARSH, *Secretary.*

SOME RECENT DEVELOPMENTS IN AIRCRAFT INSTRUMENTS

BY MAJOR H. E. WIMPERIS, M.A.

Illustrations to the Appendix of this paper were unfortunately omitted in the January number. The Appendix is therefore reprinted with the illustrations.

APPENDIX I.

Theory of A.M.L. Position Line Slide Rule

The slide rule is graduated with log cosines (outer cylinder) and log tangents (inner cylinder). The scales are graduated from 0 to 90° forwards and from 90° to 180° backwards. Any angle greater than 90° (say b) is entered or read as $180° - b$.

The following definitions and rules are printed on the rules:—

d Declination.
H Hour-angle, read in degrees from 0° to 180° East or West.
l Latitude.
c Co-latitude.
A Azimuth, read from pole of opposite name to latitude, East if H is East, West if H is West.
a Altitude.

(1) d and l same name, $Y = c + y$.
(2) d and l opposite name, $Y = c - y$.
(3) If $H < 90°$, $y < 90°$.
(4) If $H > 90°$, $y > 90°$.
(5) If $Y < 90°$, $A < 90°$.
(6) If $Y > 90°$, $A > 90°$.

From simple right angled spherical triangle theory:—

$$\tan E = \cos H \tan p.$$
$$\tan G = \sin E \tan H.$$
$$\tan G = \sin F \tan A.$$
$$\therefore \tan A \sin F = \sin E \tan H.$$
$$\therefore \tan A = \sin E \tan H / \sin F.$$

Also $\tan z = \tan F / \cos A$.

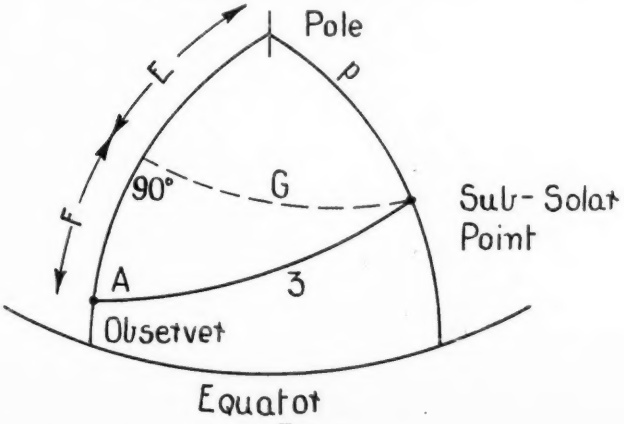


FIG. 1.

These formulæ may equally be written :—

$$\tan (90^\circ - E) = \tan d / \cos H.$$
$$\tan A = \cos (90^\circ - E) \tan H / \cos (90^\circ - F).$$
$$\tan a = \cos A \tan (90^\circ - F).$$

On the slide rule $90^\circ - E$ is called y and $90^\circ - F$ is called Y, these three formulæ becoming :—

$$\tan y = \tan d / \cos H.$$
$$\tan A = \tan H \cos y / \cos Y.$$
$$\tan a = \cos A \tan Y.$$

Proof of Rules

CASE I.—H less than 90°. d and l same name. Observer nearer pole than point where perpendicular from subsolar point meets observer's meridian (see diagram).

All sides less than 90°.

$$90^\circ - y = (90^\circ - Y) + c$$
$$Y = c + y$$

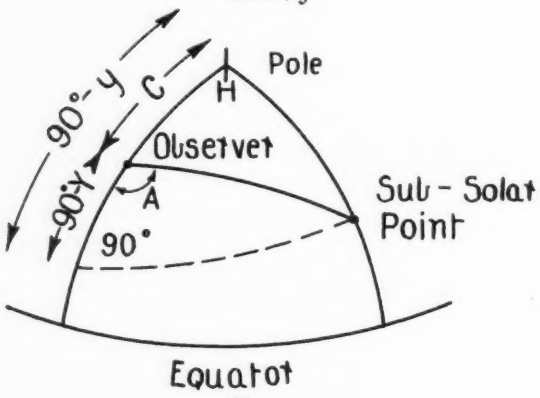


FIG. 2.

Note.—A is less than 90° and is read from pole of opposite name to latitude.

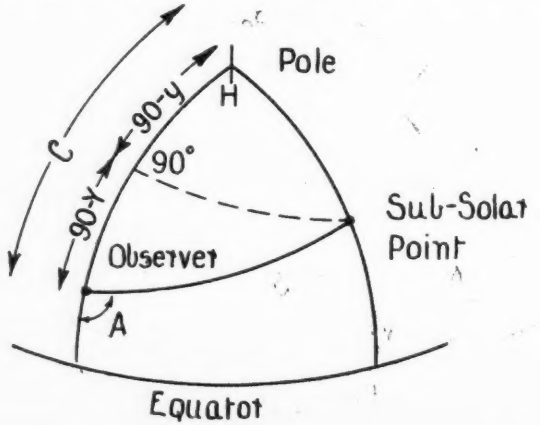


FIG. 3.

CASE II.—H less than 90°. d and l same name. Observer nearer equator than the point where the perpendicular from subsolar point meets observer's meridian (see diagram).

$$c = (90° - y) + (90° - Y)$$
$$Y = 180° - (c + y)$$

Hence Y will be entered as $(y + c)$ on the backwards scale, and under rule (6) A will be read off greater than 90°.

Note.—As A is read off greater than 90°, it is the external angle of the triangle, and is therefore measured from the pole of opposite name to latitude, this being consistent with the definition.

CASE III.—H less than 90°. d and l opposite name.

$$180° - c = 90° - Y + 90° - y$$
$$Y = c - y \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

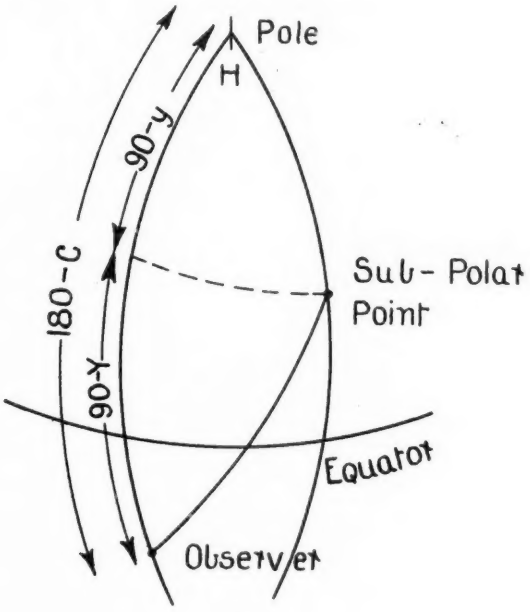


FIG. 4.

Note.—Angles H and Y are less than 90°, and A is therefore read off as the included angle of the triangle, being measured from pole of opposite name to latitude, this being consistent with the definition.

CASE IV.—H more than 90°. d and l same name.

$$90° - Y = (90° - y) + c$$
$$\therefore \quad 180° - Y = (180° - y) + c$$

As H is greater than 90°, it is entered as $180° - H$ or h on diagram.

Under rule (4) y is read greater than 90°, that is as $(180° - y)$. This makes

Y greater than 90°, and it is entered as $(180° - Y)$. Under rule (6) A is read as greater than 90° or $(180° - A')$.

Y therefore can be entered as $c + y$.

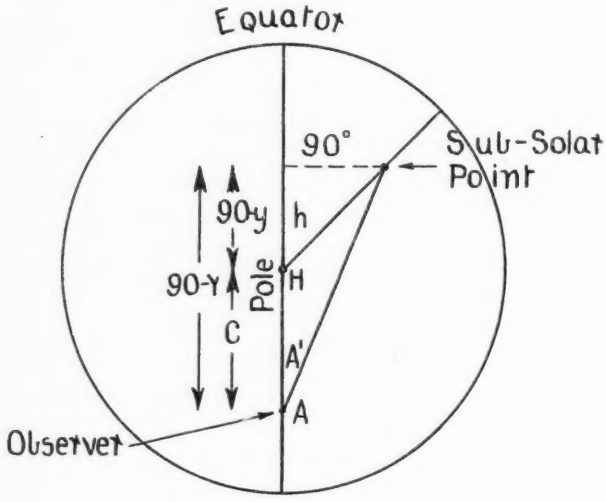


FIG. 5.

Note.—As A is read off greater than 90° it is measured from pole of opposite name to latitude, this being consistent with the definition.

PROCEEDINGS

FOURTH MEETING, 59TH SESSION

A meeting of the Society was held in the rooms of the Royal Society of Arts, John Street, Adelphi, London, on Thursday, November 15th, 1923, when a paper by Mr. H. R. Ricardo on "The Thermo-Dynamics of Aircraft Engines" was read and discussed. Colonel A. Ogilvie, the Chairman of the Society, presided.

The CHAIRMAN, in opening the proceedings, said he did not think there was any need for him to introduce Mr. Ricardo to the members in formal terms; no doubt he was well known to them as an experimental engine man and as a designer. Some of them knew his name in connection with the design of tank engines and of engines used in ordinary motor-car practice to-day. He had a research establishment at Shoreham, where all his practical and theoretical work was done, and had met with great success there. Its establishment must have been a very bold step in the first place, but extraordinarily successful results had been obtained. So far as the Society was concerned, they had to congratulate themselves on the presence that evening of someone of Mr. Ricardo's authority, and on the fact that he had taken the trouble to write a paper and read it to them. He called upon Mr. Ricardo to read his paper.

Mr. Ricardo then read his paper.

THE THERMO-DYNAMICS OF AIRCRAFT ENGINES

When asked to read a paper on the Thermodynamics of Aero Engines I felt very doubtful in the first place whether I ought to attempt such a subject at all, for, though I have posed often as a mechanic, I have not posed before as a scientist, at least not in public. In the second place I felt doubtful, if I attempted anything so rash, how to deal with the subject. Clearly I must adopt one or other of two lines, either play for safety and stick firmly to the fundamental thermo-dynamic laws which are unassailable but also inapplicable to practical working conditions, or take the plunge and endeavour to argue or at least to speculate as to how these fundamental laws must be qualified to meet actual working conditions. In adopting the latter course I realise that I must touch upon many highly controversial questions, and if in places I appear to lay down the law on what at best is little more than mere conjecture, I must ask you to forgive me and to realise from the start that the last thing I intend is to appear didactic.

The power output and efficiency of any internal combustion engine depend primarily on the ratio of expansion, since this governs the proportion of the total heat supply which can be converted into useful work on the piston.

According to the air-cycle formula, the efficiency of any engine operating on the so-called explosion cycle follows the equation $E = 1 - (1 - r)^{\gamma - 1}$. This however presupposes two conditions.

(1) That the working medium is pure dry air.
(2) That there is no interchange of heat between the air and the surrounding walls or from the previous cycle.

If only the working medium were the pure dry air so beloved of the text book, and there were no interchange of heat, the problem would be a simple one indeed, but unfortunately the engine would not work. We must add to the air some hydrocarbon to burn up its oxygen and we have interchange of heat not only with the walls of the cylinder but also with the products of the previous

cycle, all of which complicate the issue. We have also to deal with such factors as change of specific heat, dissociation, detonation, rate of burning, etc.

According to the air cycle, the efficiency is dependent on the expansion ratio alone, and is independent of temperature, but when account is taken of the increase in specific heat of the working fluid at high temperatures, of dissociation, and of the loss of heat to the cylinder walls, this formula requires considerable modification.

Specific Heat

Some years ago the Gaseous Explosions Committee of the British Association carried out a very careful investigation into the change of specific heat of the products of combustion of a gas/air mixture; these determinations, however, were not applicable to petrol/air mixtures. More recently Tizard and Pye have made a very careful analysis of the change of specific heat and dissociation of petrol, benzol, and alcohol mixtures at the very high temperatures prevailing in an engine cylinder. Though there is some reason to suspect that their figures slightly over-estimate the change of specific heat, they must certainly be very near the mark.

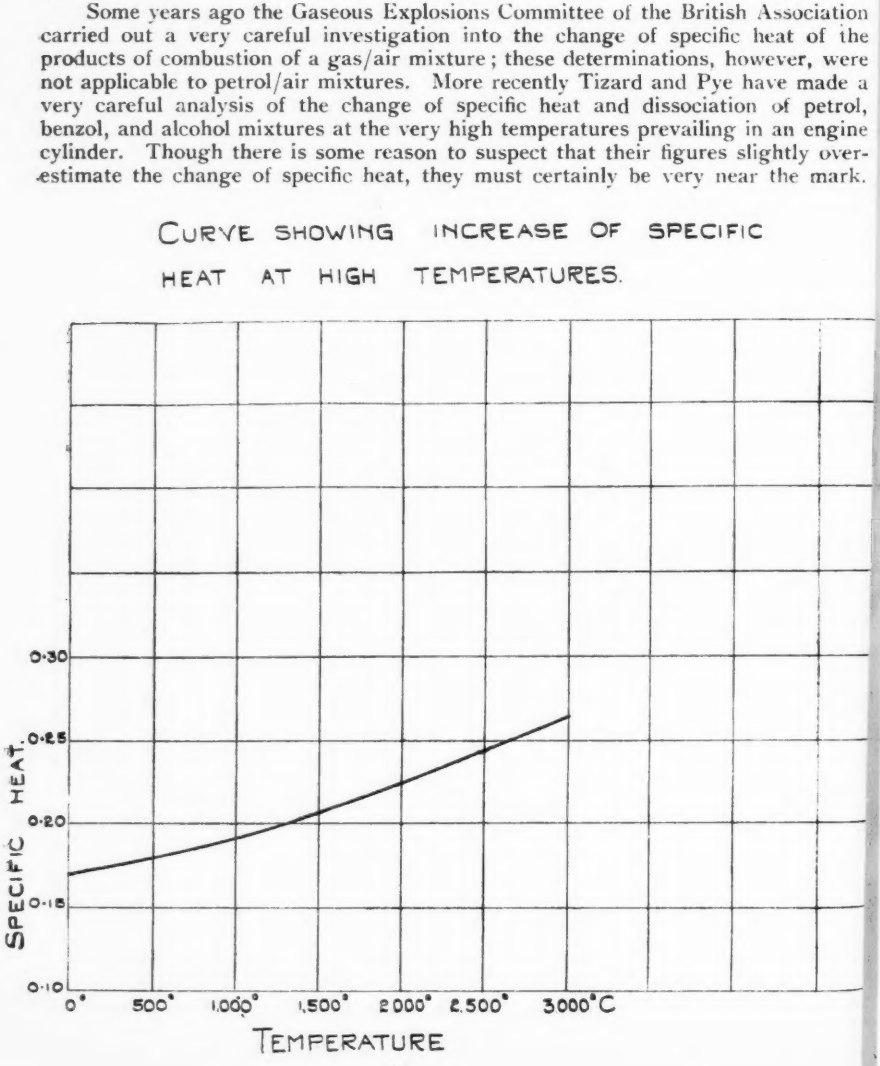


FIG. 1.

The increase of specific heat at high temperatures represents a dead loss as compared with the air cycle basis, since it means, in effect, that any heat added at high temperatures is not accompanied by so large an increase in pressure as at the lower temperatures. At the temperatures prevailing in an aero engine or indeed in any petrol engine, namely, between 2,500°C. and 1,800°C., the loss due to change of specific heat is of a very large order and constitutes the greatest discrepancy between the practically possible and the air-cycle ideal. Tizard and Pye have shown that change of specific heat is greatest in fuels containing a large proportion of hydrogen rather than carbon, that is to say, on this account alone and for equal maximum temperatures the power output obtainable on petrol would be less than on benzole, did not other factors intervene to restore the balance.

From the curve (Fig. 1) which shows the change in specific heat of heptane (petrol) with temperature it will be seen how great is the change and how great the importance of keeping the maximum temperature as low as possible.

Dissociation of the products of combustion at high temperatures, that is, the breaking down of CO_2 into CO and O, and H_2O into free hydrogen and oxygen, with consequent absorption of heat, is another factor which has to be taken into account. Fortunately it does not play such havoc with the power and efficiency as change of specific heat, because it occurs to any appreciable extent only at very high temperatures, so that as the expansion proceeds and the temperatures fall,

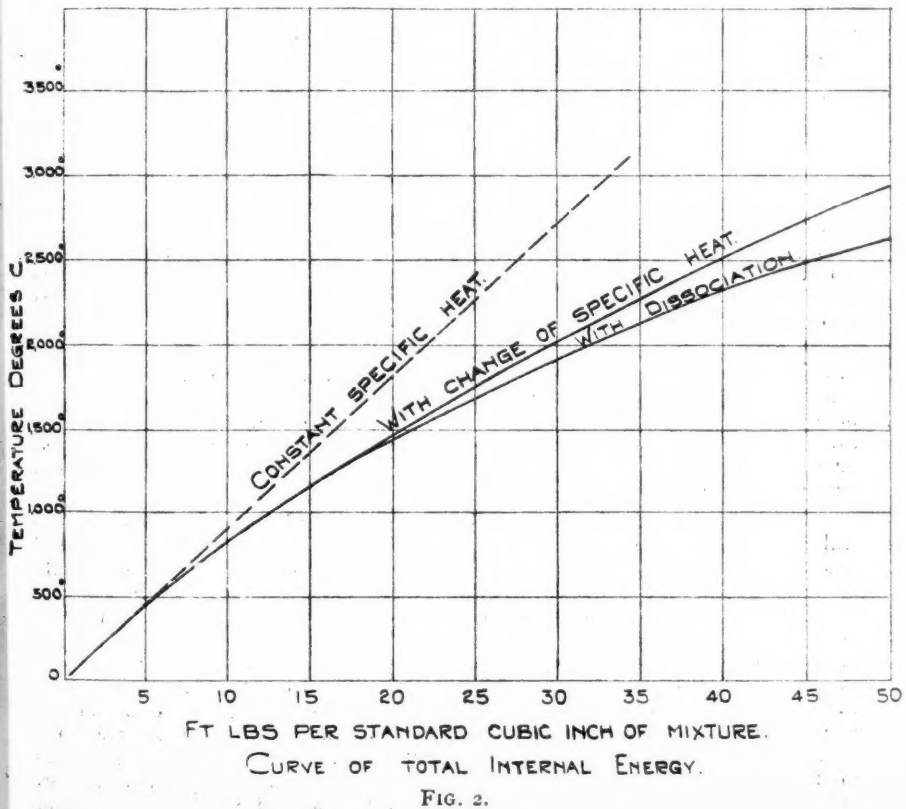


FIG. 2.

re-combination with corresponding liberation of heat takes place and a part, at any rate, of the lost heat is recovered, and that in time for some, though not the full use, to be made of it. Dissociation has, however, an important bearing in that its adverse effects are reduced as the ratio of expansion is increased, for not only does the higher pressure reduce the amount of dissociation (at the same temperature) but the more prolonged expansion permits of more use being made of the heat liberated by re-combination, with the result that as we increase the expansion, or compression ratio as it is more usually termed, we get a higher efficiency than the air cycle formula would lead us to expect. As in the case of the change of specific heat the ill effects of dissociation would be greatly reduced if only we could work at a lower temperature. The curve (Fig. 2) shows the total internal energy available at different temperatures after taking into account change of specific heat and dissociation. The dotted line shows the internal energy with no change of specific heat or dissociation.

Heat Loss to the Jackets is yet another factor which prevents our attaining to the ideal air cycle efficiency. It is not, however, of anything like the magnitude that many people believe. Of all the heat disposed of by the jacket water or radiating fins by far the bulk is waste heat in any event, and only a small amount, namely about 40 per cent., of that lost during combustion and a much smaller percentage of that lost during expansion could, in any event, be turned into useful work. Yet again, the lowering of maximum temperature due to heat loss during the period of combustion tends to reduce the loss due to change of specific heat and dissociation, so that even at this critical stage there is some compensation. If in an aero engine of good modern design the entire flow of heat to the jackets were suppressed, the net gain in power and efficiency would be of the order of about 8 per cent. only. I do not mean to imply that heat loss to the cylinder walls is of no account, but it certainly is not anything like so serious as many suppose.

Against these sources of loss we have to offset one advantage, namely, the increase in the volume of the products of combustion of all the volatile liquid fuels. This varies with the nature of the fuel, ranging from about 6 per cent. with alcohol and 5 per cent. with petrol down to about 2 per cent. with benzol. The smaller volume increase with benzol just about balances the smaller loss due to change of specific heat, so that the power output and efficiency obtainable from either of these fuels is almost exactly the same.

It is clear that all the three main sources of loss to which I have referred are directly dependent upon the flame temperature, and that if, by any means, we can reduce this temperature we shall at once gain in efficiency; nor is that all, for nearly 90 per cent. of present-day engine troubles are due directly or indirectly to excessive heat flow. Directly in the form of burnt out valves, sparking plugs and pistons; indirectly in the form of carbonised oil, gummed-up piston rings and defective cylinder lubrication.

The aggregate losses from change of specific heat, direct heat loss to the walls and dissociation, reduce the possible efficiency with petrol or benzol mixtures to approximately 70 per cent. of the air cycle efficiency. If means could be found to reduce the flame temperature, the efficiency would rise in a nearly straight line from 70 per cent. to 100 per cent. as the temperature is reduced. So long as a homogeneous mixture of petrol vapour and air is used it is necessary, in order to get sufficiently rapid burning, to work at all times at or very near the maximum flame temperature; any attempt at reducing the temperature more than about 10 per cent. by dilution with excess air or inert gases results merely in slow and incomplete combustion with consequent loss of efficiency and back firing. With all such fuels as petrol or benzol, etc., we need flame temperatures of the order of 2,300 deg. C. as a minimum, to obtain sufficiently rapid burning, but with hydrogen this is not the case, for with this fuel and, so far as is known at

present, with this fuel alone is it practicable to work with very diluted mixtures until the flame temperature is reduced to one half or less, and as a result, not only does it become possible to realise the very high theoretical efficiency obtainable from the low flame temperature with hydrogen, but the presence of even a small whiff of hydrogen will, by its rapid burning, serve to enable very weak petrol/air mixtures to be used, such as could not ordinarily be ignited at all. In this way, by using a small priming of hydrogen, one can work with petrol or even kerosene at an efficiency quite unobtainable by ordinary means, namely, about 42 per cent. or even higher, and, at the same time, reap all the mechanical advantages from low flame temperature such as cool valves, absence of carbonisation, etc. Unfortunately, apart from airship service, the use of hydrogen as a priming charge is outside of practical politics, and other means must be found for working at a low temperature.

Interchange of Heat

We come next to the question of interchange of heat between the residual products of the last cycle and the fresh mixture. It has always been assumed in all calculations, etc., that the whole of the heat contained in the residual exhaust products is imparted to the incoming charge, with the result that the former contract and the latter expands in proportion, so that (if there is no wire-drawing) the volume of the incoming charge at its entering temperature will be equal to the swept volume, regardless of the quantity or temperature of the residual exhaust products, provided that their pressure is atmospheric. According to this convenient assumption the volumetric efficiency of an engine is independent of its compression ratio. Our experiments on a variable compression engine showed that the volumetric efficiency fell very appreciably with increase of compression, and it was long before an explanation could be found, though, when found, it appeared simple and plausible enough. Let us suppose that at the end of the exhaust stroke the piston stopped with both exhaust and inlet valves closed. Under these conditions the entrapped exhaust products would soon lose their heat to the walls and would contract to about one-quarter of their volume. Now suppose that the inlet valves opened and the piston descended; there would then be no interchange of heat and the volume of mixture would, at its entering temperature, be equal to the whole of the swept volume plus three-quarters of the clearance volume, instead of equal to the swept volume only. This is only a gross exaggeration of what no doubt actually does happen. After the completion of the exhaust stroke and during the early period of the suction stroke, the exhaust products are, of course, losing heat rapidly to the cylinder walls, and the volume of mixture entering is therefore always greater than the swept volume (exclusive, of course, of wire-drawing), how much greater depends upon the proportion and temperature of the exhaust products, the greater the proportion and the hotter the products, *i.e.*, the lower the compression, the higher will be the volumetric efficiency, as indeed we have found. This conclusion suggests that it would pay to prolong the time interval between the closing of the exhaust and the opening of the inlet valves, but here we are faced with the difficulty that the period of opening of the inlet valve in a high-speed engine is already as short as mechanical considerations will permit. This difficulty can, however, be overcome by masking the inlet valves during a portion of their travel. The value of absorbing the heat of the products of the previous cycle suggests other ways of accomplishing the same end, which I will deal with later.

Influence of Latent Heat

The influence of the latent heat of evaporation of a fuel is one which is often overlooked, yet it is the one characteristic above all else which determines the power output available from an engine. All mixtures containing hydrocarbon fuel

vapours have, within less than 2 per cent., the same total internal energy, but their latent heat of evaporation differs, and, since the latent heat of the fuel, like the loss of heat by the exhaust products, serves to lower the suction temperature and so increase the weight of charge taken into the cylinder, it follows that the fuel with the highest latent heat will give the highest power output, provided it is volatile enough. Among hydrocarbon fuels, such as petrol or benzol, the latent heat varies comparatively little, and such small variations as occur just balance any slight variation in internal energy, so that at any given compression ratio, provided there is no detonation or incipient detonation, all hydrocarbon fuels yield the same power output to within less than 1 per cent. In the case of alcohol, however, the latent heat is relatively great, and although the total internal energy of an alcohol vapour and air mixture is some 2½ per cent. less than petrol, yet, on account of its higher latent heat, the weight of charge taken into the cylinder is nearly 10 per cent., and the power output about 7·5 per cent. greater than with petrol.

It must be remembered that in any engine working on the explosion cycle the whole of the heat required to evaporate the fuel is supplied from the waste products of the previous cycle or from the hot cylinder walls, etc., whereas in the case of the compression ignition engine it has all to be supplied from the heat of combustion of the fuel. This puts the compression ignition engine on an inferior footing as regards power and efficiency. In the case of hydrocarbon fuels, however, the latent heat of evaporation is less than 1 per cent. of the heat of combustion so that the disadvantage is small, in respect to the other advantages of this cycle, but in the case of alcohol, the latent heat represents about 4 per cent. of the heat of combustion, so that it would be sheer madness to build compression ignition engines to run on alcohol, or indeed on any fuel with a high latent heat of evaporation.

There is another point in relation to the latent heat of a fuel to which the writer would like to call attention. All or nearly all determinations of calorific value are made by burning the liquid fuel in a bomb, in which case the latent heat of the liquid has to be supplied from the heat of combustion; applied to explosion engines, such determinations are misleading, for, as mentioned previously, in such engines the latent heat of the liquid is supplied by waste heat. Applied to explosion engines, therefore, the latent heat of evaporation should be added to the calorific value. Unless such a correction is made the thermal efficiency will appear higher than is, in fact, the case, and it is to lack of appreciation of this point that the absurdly high thermal efficiencies recorded in certain Government reports of experiments with alcohol are due.

Pre-Burning

There is yet another factor which has come into prominence lately and which plays a highly important part in governing the power output and efficiency of an engine, namely, the effect of what, in lieu of a better term, may be called pre-burning of the mixture during the suction and compression strokes. As the gases circulate over surfaces whose temperature is far above their so-called self-ignition temperature a certain amount of surface combustion evidently takes place. The temperature of this slow surface combustion is not nearly high enough to permit it to spread throughout the whole charge in the time available, but it may, and no doubt often does, spread throughout quite an appreciable portion of it, sufficient both to pull down the power and efficiency and at the same time to raise the temperature and so increase the tendency to detonate. It is difficult to prove convincingly the presence of this pre-burning, though there can be little doubt as to its existence; it is still more difficult to assess the extent of it, which is controlled clearly by both the time element and the temperature of the surface. In the

ordinary engine cylinder the only surface which is likely to cause pre-burning is the head of the exhaust valve, and the extent of pre-burning from this surface probably varies as something like the cube of its temperature. If means could be found for cooling the exhaust valve or, better still, abolishing it, we should improve both our power output and efficiency quite appreciably, as has been shown by experiments with water-cooled valves, with sleeve valves, and even to quite a marked degree when particular care has been devoted to the cooling of exhaust valve seatings.

Flame Temperature and Range of Burning

We come now to the very vexed question as to the actual process of combustion in an engine cylinder, and here, in the absence of more precise information, we are forced to fall back on speculation. The theoretical cycle we work on assumes that combustion is instantaneous and at constant volume; in practice, of course, it is neither, nor would either be desirable. In the first place,

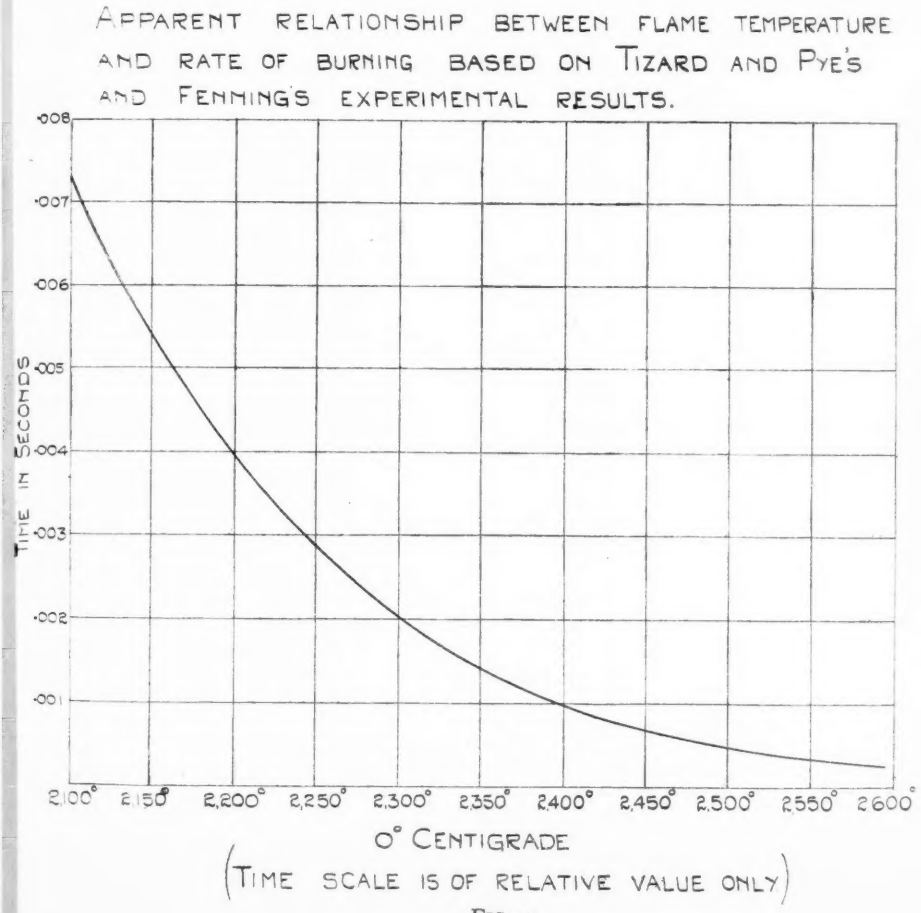


FIG. 3.

instantaneous or even extremely rapid combustion gives rise to severe shock, and, in the second, we do not, on account of heat loss, want to attain maximum temperature one moment before we need. The piston of an engine may be regarded as virtually stationary for, say, 10 deg. on either side of top centre, and we most certainly do not want the working fluid lying trapped at its maximum temperature during this virtually idle period. In practice we save much in heat loss, and lose practically nothing in expansion, if we arrange for maximum pressure to be reached from 10 deg. to 15 deg. after top centre and maximum temperature somewhat later still.

What appears to happen is substantially as follows :—A single intensely high temperature spark passes across the electrodes, leaving behind it a thin thread of flame. From this thin thread combustion spreads to the envelope of mixture immediately surrounding it at a rate which depends primarily upon the temperature of the flame front itself and to a secondary degree upon both the temperature and the density of the surrounding envelope. In this manner there grows up, gradually at first, but at a rapidly increasing rate, a small nucleus of flame. If the contents of the cylinder were at rest, as in the case of an explosion vessel, this process would spread with increasing speed until it extended throughout the whole mass, but if, at any period, the rate of propagation exceeded a certain limiting figure, depending on the nature of the fuel, a detonation wave would be

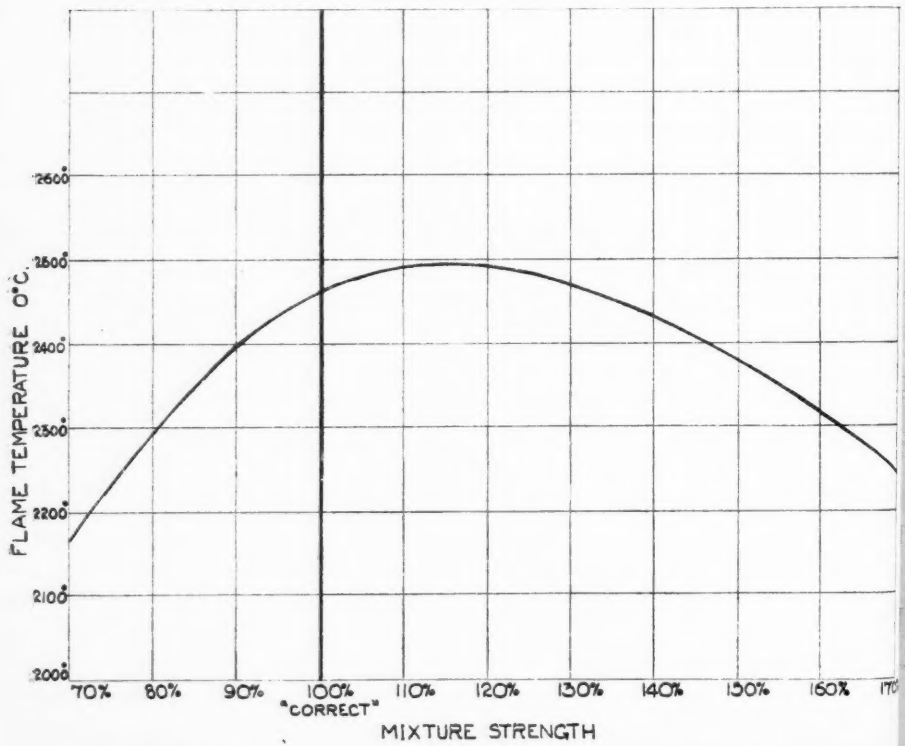


FIG. 4.

set up. In the actual engine cylinder, however, the mixture is not at rest but is being whirled about very rapidly, and is, in fact, in a highly turbulent condition. So soon, therefore, as a self-propagating nucleus of flame has been formed, it is torn into fragments, which are spread and whirled about throughout the whole mass, with the result that the combustion process is speeded up enormously, as it were by handing round the fiery touch. Left to itself, the combustion process would be far too slow to be of any use in an engine cylinder, and though the flame temperature and rate of burning might be such as to cause detonation during the latter stages of the process, yet the process as a whole would be too slow even for an engine running at only 200 r.p.m. We rely therefore entirely on turbulence, without which no internal combustion engine could run. As a mental picture the writer prefers to regard the process as though it developed in two quite distinct stages, one the growth and development of a self-propagating nucleus of flame and the other the distribution of this flame throughout the combustion chamber. The former is a chemical process depending upon the nature of the fuel, upon both temperature and pressure and also upon what Tizard terms the temperature coefficient of the fuel, that is the relationship

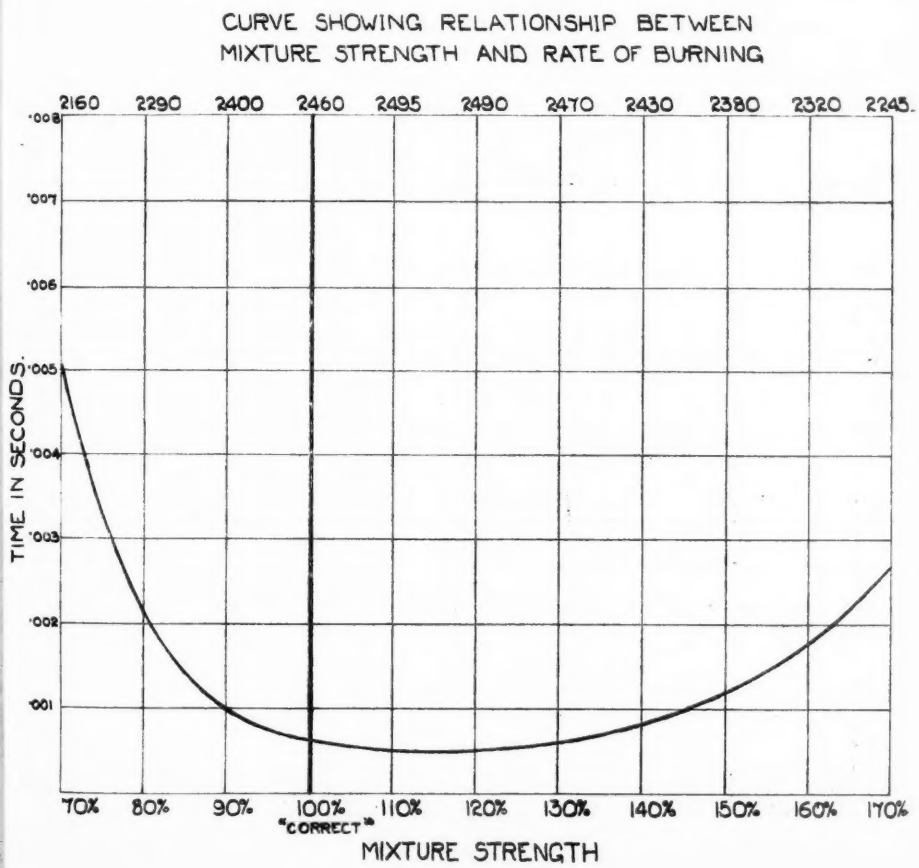


FIG. 5.

between flame temperature and rate of acceleration. The second stage is a mechanical one pure and simple. I have suggested that these two stages are separate and distinct, but they must, of course, interact upon one another to some extent, for example the higher the flame temperature and the more rapid the rate of burning, the more rapidly will combustion spread with a given rate of turbulence.

Quite recently two long series of experiments have been carried out, by Fenning at the National Physical Laboratory, and by Tizard and Pye at the writer's laboratory. Fenning used a cylindrical explosion vessel 7in. dia. by 8in. long and worked with a stagnant charge ignited electrically. Tizard and Pye used a cylinder of 4½in. dia., in which the mixture was compressed adiabatically and was ignited by the heat of compression; in their case the charge was also stagnant, except in so far as it was disturbed by rapid compression, but in some few of the experiments, turbulence was produced by means of a small fan rotating inside the cylinder. I will refer to these experiments later, but for the present it is sufficient to say that the results of both are applicable to the first stage of combustion according to the general theory I have been discussing, but when one tries to express the rate of burning in absolute terms, the results are a little conflicting. Tizard and Pye find that, other things being equal, the rate of burning trebles for every 4 per cent. increase in flame temperature. Fenning's

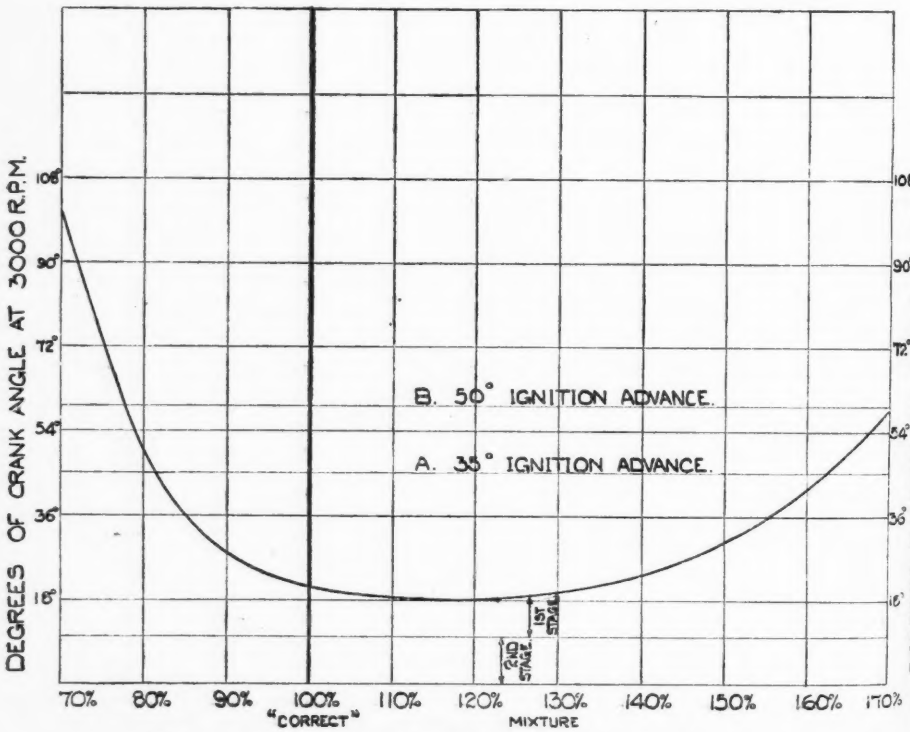


FIG. 6.

results would seem to show a somewhat lower rate, while a figure nearly midway between these two would appear to fit the writer's own observations from actual engine tests. The curve (Fig. 3) shows the writer's suggested relationship between flame temperature and rate of burning. The figures on the vertical scale represent the time from the passage of the spark to the formation of a self-propagating nucleus of flame, that is to say, they represent the time occupied by the first stage of the combustion process. Needless to say, they must not be regarded as hard and fast values, but only as indications of a general relationship. The curve (Fig. 4) shows the flame temperature at different mixture strengths assuming a compression ratio of 5 : 1 and taking into account all such considerations as dilution with the residual exhaust products, heat given up to the incoming charge, dissociation, etc. This curve is probably correct for petrol mixtures to within about 2 or 3 per cent. The curve (Fig. 5) shows the two previous curves expressed in terms of rate of burning against mixture strength. In the curve (Fig. 6) the above has been expressed in terms of degrees of crank-angle of an engine running at 3,000 r.p.m., and a constant time, namely, 10 deg. of crank-angle, has been added throughout to represent the distribution by turbulence—that is the second stage. This curve, therefore, expresses the time from the passage of the spark to the attainment of maximum pressure. This period is, in practice, limited to 60 deg. at the very outside and probably to about 45 deg. for efficient working, corresponding to 50 deg. and 35 deg. of ignition advance, for maximum pressure

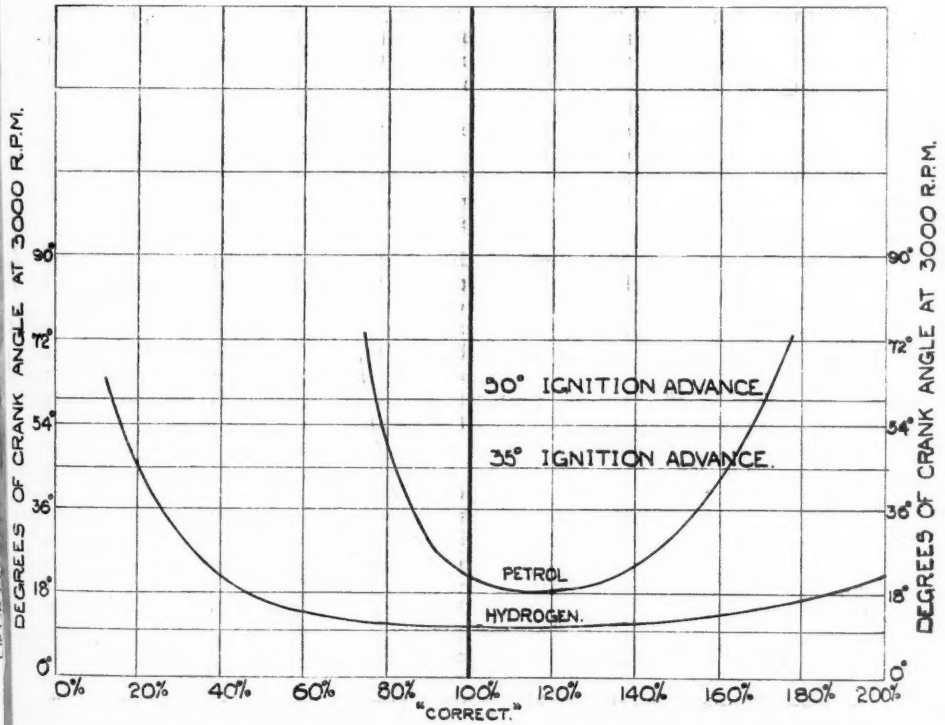


FIG. 7.

should not be attained earlier than 10 deg. after top centre. The horizontal lines *A* and *B* show, therefore, the range of mixture possible with maxima of 35 deg. and 50 deg. ignition advance respectively. So far as all volatile liquid fuels are concerned, the flame temperature and range of burning does not vary very greatly, and we are compelled to work with a flame temperature exceeding about 2,250 deg. C. At the other end of the scale if we exceed about 2,500 deg. C. the rate of burning becomes so great as, in the case of most fuels, to induce detonation, hence in practice we can work only between a very restricted range of flame temperature. Some while ago we carried out a series of experiments on behalf of the Air Ministry, in which we raised or lowered the flame temperature on the one hand by the addition of oxygen, on the other of inert diluents, such as nitrogen, water-vapour, etc. These proved that by no such means could the range of flame temperature be extended—additional oxygen and a higher flame temperature gave rise at once to excessive detonation, while additional nitrogen merely prevented the petrol/air mixture being weakened so that the flame temperature could not be reduced below about 2,250 deg. C. The addition of inert gases did, however, tend to suppress detonation (by lowering the maximum flame temperature), and permitted a considerable increase in compression ratio, and therefore in efficiency. Curiously enough, in all cases, detonation occurred when a certain maximum pressure was reached, irrespective of whether it was attained by a dilute oxygen charge at high compression, an enriched charge at low compression, or a normal charge. This may be and probably is a pure coincidence, but if so it is a sufficiently striking one to be well worth recording. In the case of hydrogen the rate of burning is something like twelve times as great as petrol. If now we alter the above curve by increasing the speed of the first stage twelve times we arrive at the result shown in Fig. 7, from which it will be seen that with the same limiting ignition advance we can reduce the limit of mixture weakness from about 82 per cent. of the fuel required for complete combustion to only about 20 per cent., and that we can, in fact, work with a very weak mixture indeed at a correspondingly low flame temperature, and so derive all the benefits to be expected from it; a plausible explanation can thus be found for the apparently remarkable behaviour of hydrogen.

In support of the above general theory the following experimental results may be cited. Fig. 8 shows a number of diagrams taken by Fenning during the very beautiful and finished series of experiments to which I have already alluded. They show the effect of mixture strength and therefore of flame temperature on the rate of burning of a stagnant air charge and, in the last example, it will be seen that the rate is sufficient to set up detonation just before the completion of combustion. In all cases the temperature and pressure of the mixture before ignition were constant. Fig. 9 shows two diagrams in which the same mixture strength was used and at nearly the same initial pressure, but at an initial temperature in one case of 100 deg. C., and in the other of 201 deg. C., showing quicker burning and a tendency to detonate at the higher temperature. So far as I can deduce, however, the increased rate of burning in this case is only such as could be accounted for by the higher flame temperature, and need have no other relation to the initial temperature, for an increase of 101 deg. C. in the initial temperature means a similar increase in the flame temperature, which alone appears sufficient to account for the difference in the rate of burning. Fig. 10 shows two comparative diagrams at the same mixture strength, the same initial temperature and the same flame temperature, but at different initial pressures, showing, it must be admitted, apparently much the same rate of burning, though at the slightly higher initial pressure extremely violent detonation takes place.

The diagrams (Fig. 11) show the relative rates of burning of complete combustion mixtures of hydrogen and petrol. They are not strictly comparable, for the

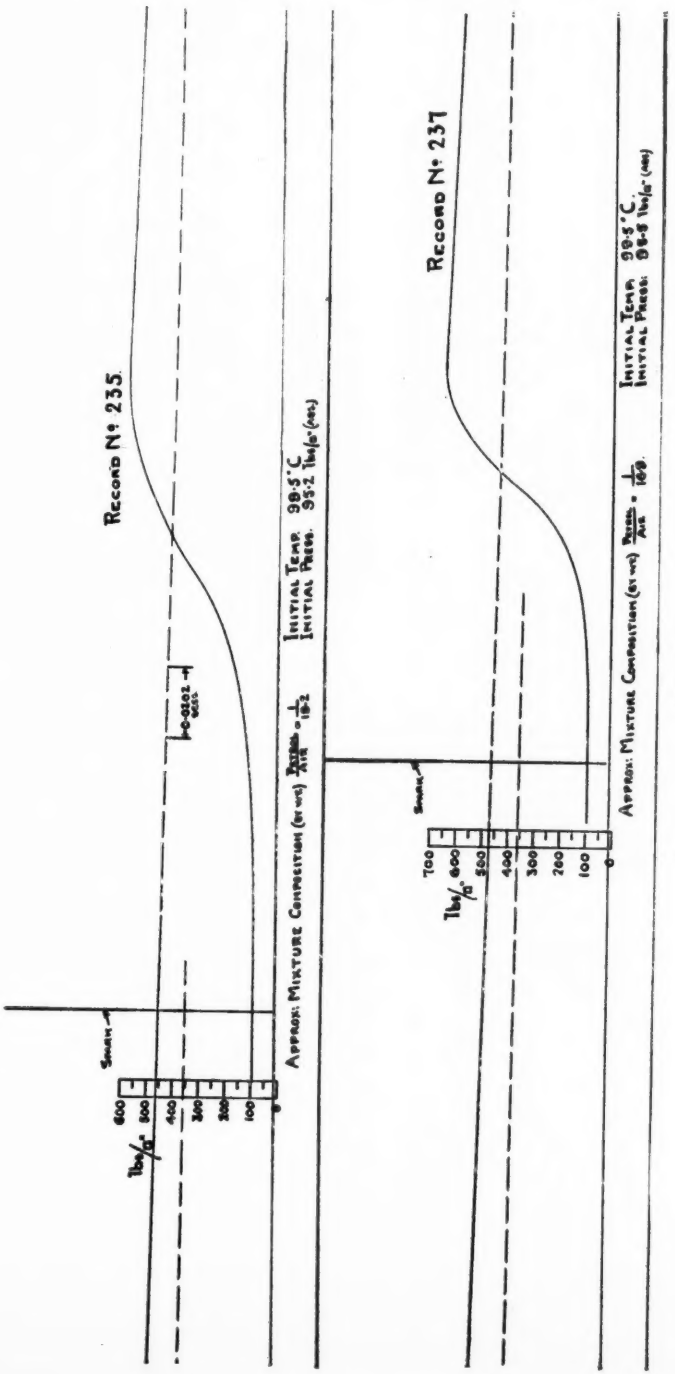

RECORD Nº 235.
RECORD Nº 237
SMOKE
600
500
400
300
200
100
0
lbs/in²
APPROX: MIXTURE COMPOSITION (BY WT) Petrol/Air = 1/18·2
INITIAL TEMP. 99·5°C
INITIAL PRESS. 93·2 lbs/in² (abs)
APPROX: MIXTURE COMPOSITION (BY WT) Petrol/Air = 1/16·6
INITIAL TEMP. 99·5°C
INITIAL PRESS. 95·8 lbs/in² (abs)

FIG. 8.

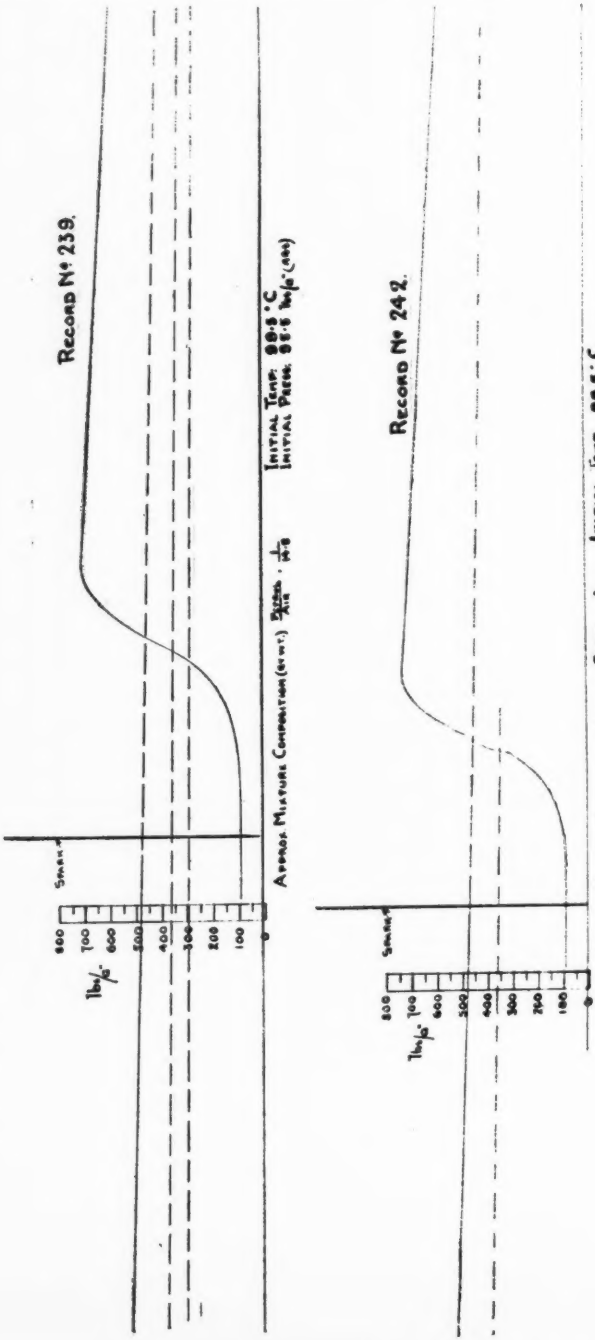


FIG. 8A.

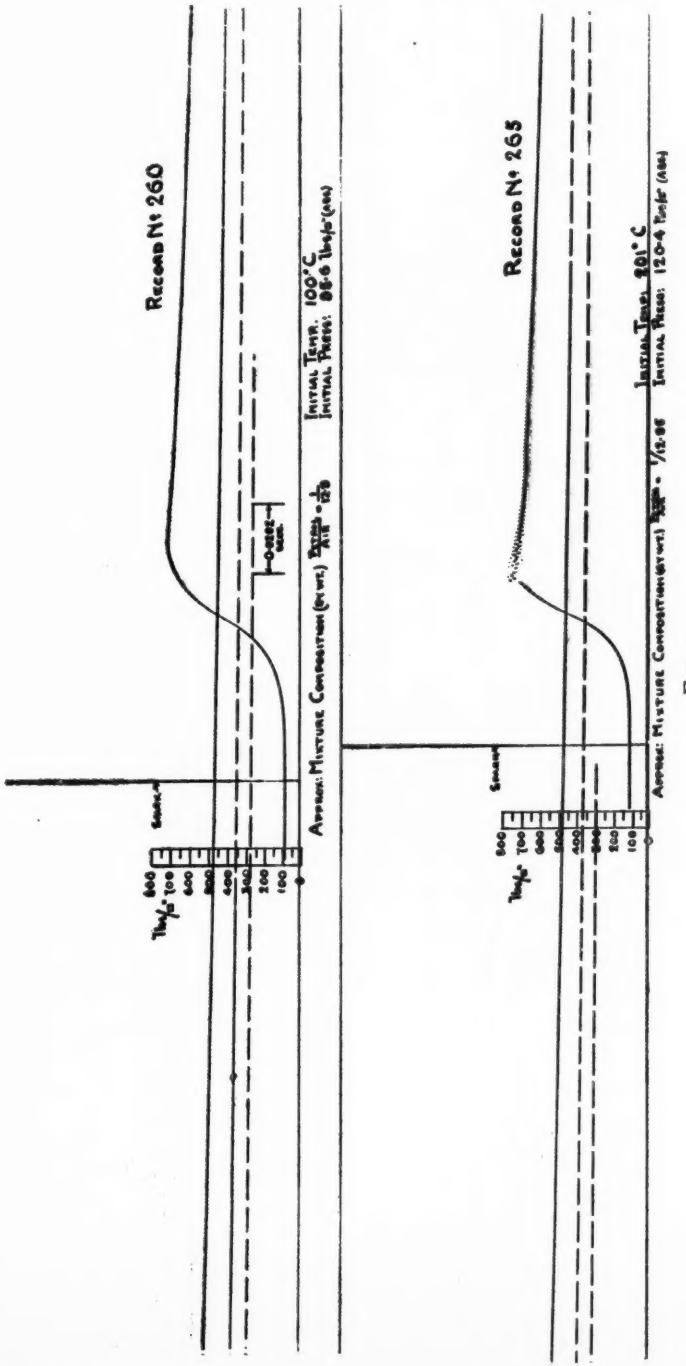


FIG. 9.

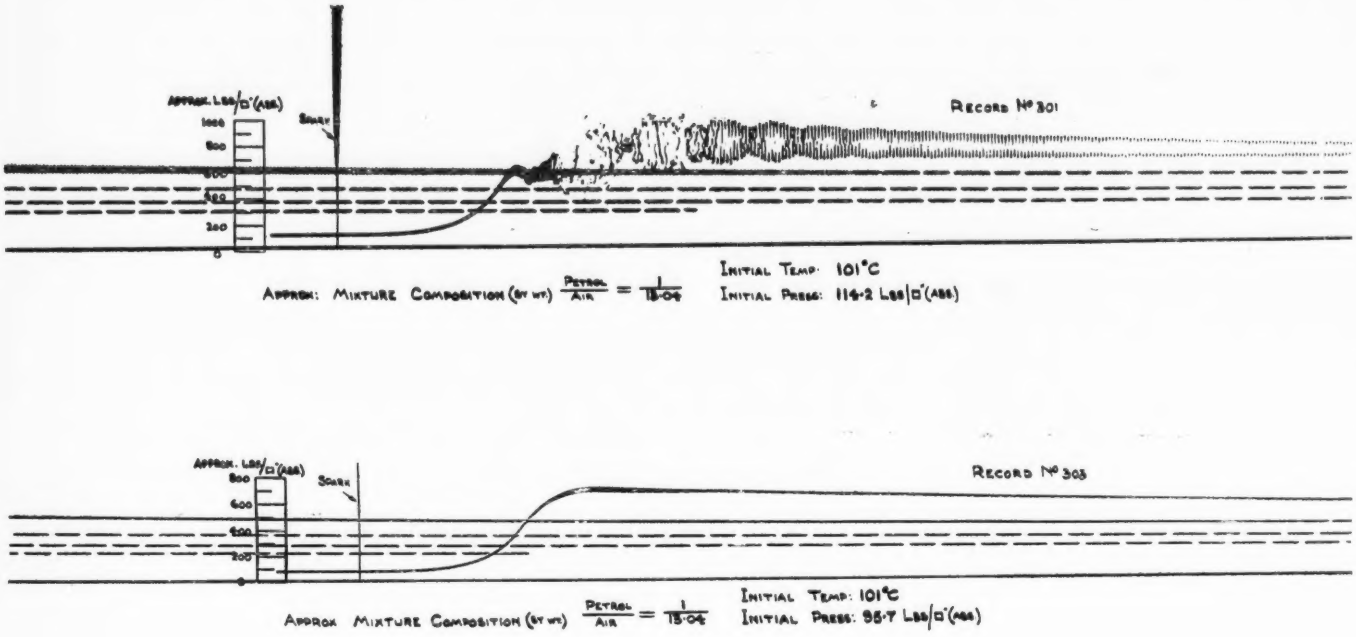

APPROX. LBS/□"(ABS)
1000
800
600
400
200
0
SPARK
RECORD No 301
APPROX: MIXTURE COMPOSITION (BY WT) PETROL/AIR = 1/15·04
INITIAL TEMP: 101°C
INITIAL PRESS: 114·2 LBS/□"(ABS)
APPROX. LBS/□"(ABS)
800
600
400
200
0
SPARK
RECORD No 303
APPROX MIXTURE COMPOSITION (BY WT) PETROL/AIR = 1/15·04
INITIAL TEMP: 101°C
INITIAL PRESS: 98·7 LBS/□"(ABS)

FIG. 10.

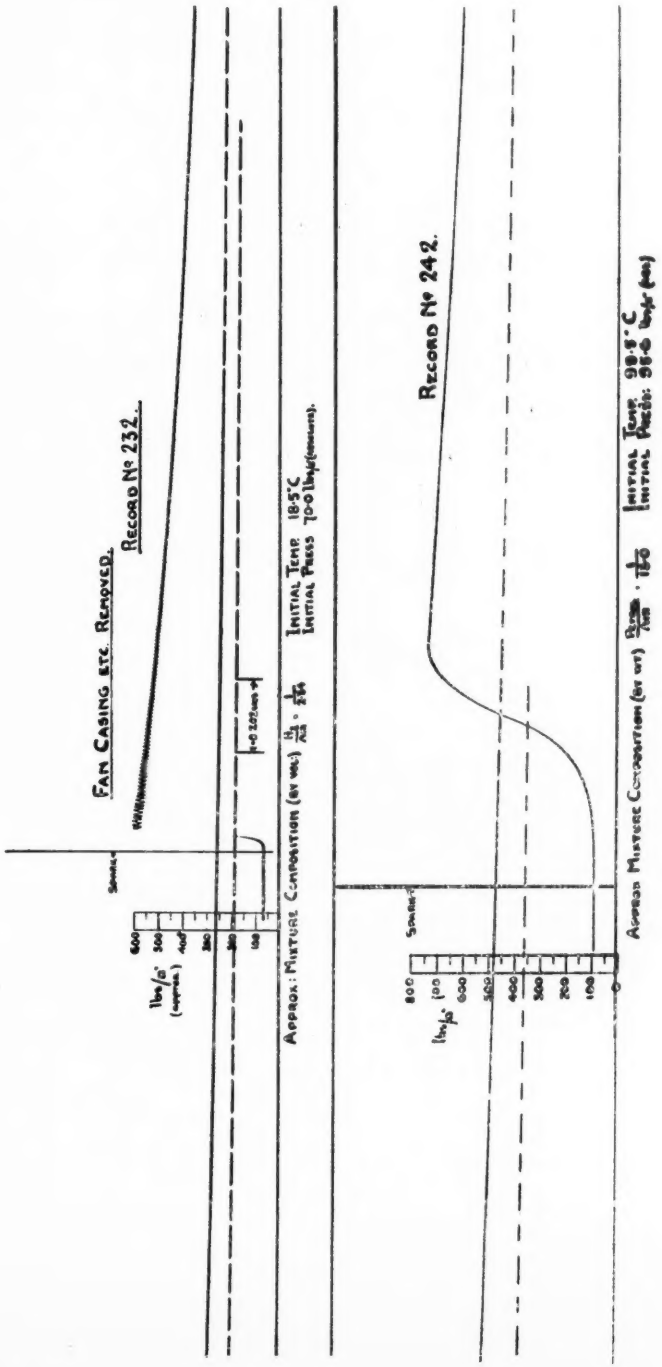

FAN CASING ETC. REMOVED.
RECORD Nº 232.
RECORD Nº 242.
SPARKS
lbs/in² (approx.)
600
500
400
300
200
100
0
800
700
600
500
400
300
200
100
0
APPROX. MIXTURE COMPOSITION (BY VOL.) 1/24 · 1/25
INITIAL TEMP 18.5°C
INITIAL PRESS 70·0 lbs/in² (approx.)
APPROX MIXTURE COMPOSITION (BY WT.) 1/15 · 1/16
INITIAL TEMP 98.5°C
INITIAL PRESS: 98·0 lbs/in² (approx)

FIG. 11.

initial temperature in the case of hydrogen was atmospheric only, while in the case of petrol the mixture was heated to 100 deg. C., so that the contrast should really be even more marked.

The diagram (Fig. 12) is a typical example from Tizard and Pye's experiments

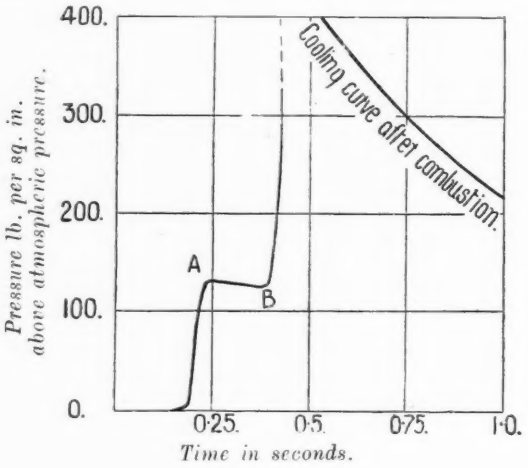


FIG. 12.

Indicator diagram of ignition of Heptane-air mixture.

Maximum compression temperature 310 deg. C.

Delay=0.17 seconds. Fan full speed.

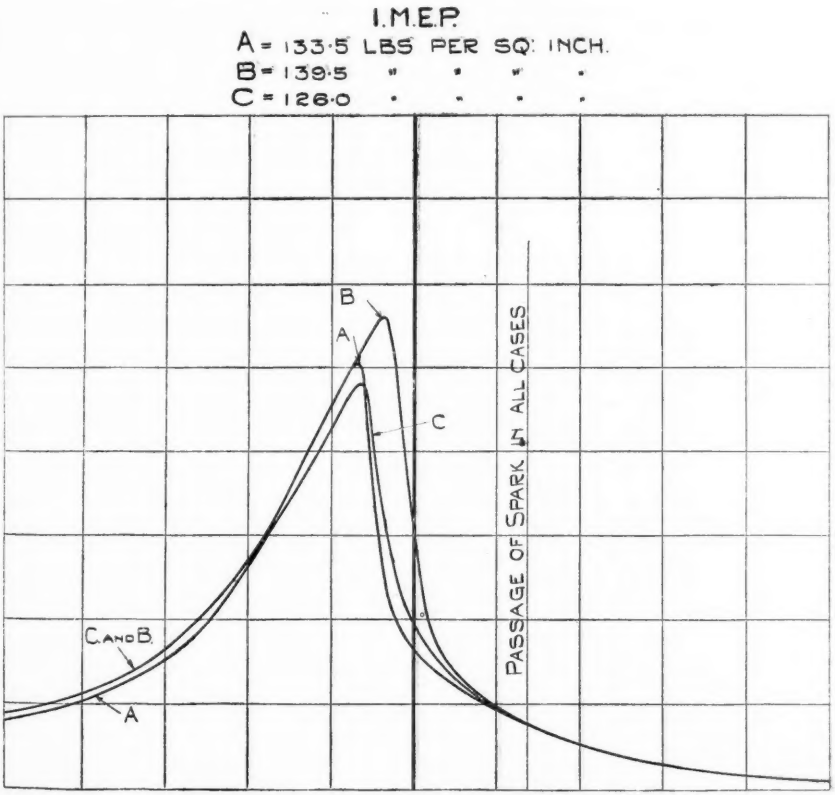
with ignition brought about by compression alone; here again, as in the case of electrical ignition, burning evidently starts from one spot, spreads slowly at first, and develops into detonation later on.

The curves (Fig. 13) are actual indicator diagrams from an experimental aero engine running at constant temperature but with three different mixture strengths; they are all taken with that very beautiful instrument the R.A.E. indicator. The curve (Fig. 14) shows the ignition timing required at different mixture strengths in order to develop maximum pressure 10 deg. after top centre. All the above experimental results relate to what may be termed the first stage in the process of combustion.

The second stage depends upon turbulence. We rely upon turbulence for two essential functions, firstly to spread inflammation and, secondly, to scour away the stagnant layer of mixture which, clinging closely to the cold cylinder walls, can get rid of its heat so rapidly as either to escape combustion altogether or to burn so late in the expansion stroke as to be of very little value. It can be demonstrated that the very wide differences in performance as between, say, an overhead-valve engine and one with valves in side pockets is to be accounted for almost entirely by the different degree and formation of turbulence in the two cases, and that it is not due, except in very small part, to heat losses, volumetric efficiency or any of the causes to which it is generally attributed. In the ordinary types of engine, turbulence is set up by and depends upon the initial velocity through the inlet valves; we need all the turbulence we can possibly get, and we need it, too, in the form of a general swirl, as distinct from local eddies. Since turbulence depends upon inlet velocity, it follows that it will be nearly proportional

to engine speed, and the time taken to spread inflammation will therefore also be nearly proportional, so that in terms of crank-angle the spread of inflammation will be constant. I have said that we want all the turbulence we can get from inlet velocity, and we could do with much more yet, but it is also possible to have excessive turbulence, when the normal inlet velocity is supplemented by other means. In the case of poppet valve engines, and so long as we rely upon the velocity through the inlet valves, we cannot get turbulence enough without excessive wire-drawing, but in the case of sleeve valve engines, owing to the more efficient orifice coefficient of the ports and to the fact that the mixture enters the cylinder as a solid stream as compared with a hollow cone, we can get all the turbulence we want and, if we are not careful, too much. As we increase turbulence :—

(1) We scour away and bring into use the stagnant layer of gas adhering to the walls;
(2) We speed up the rate of inflammation, but
(3) We increase the heat loss to the walls; and
(4) After a certain point we tend to reduce the possible range of burning.



THREE SUPERIMPOSED DIAGRAMS.

A MIXTURE 17·2% B MIXTURE 12·5% C MIXTURE 8·9·5%

FIG 13.

Experiments which the writer has carried out recently indicate that, for each fuel, there is an optimum degree of turbulence beyond which any further disturbance results merely in excessive heat loss and a narrowing of the possible mixture range, while he has succeeded in reaching such a degree of turbulence as to prevent ignition altogether, presumably by the dissipation of the nucleus of flame before it can become fully established at the plug points; in other words, the draught may be such as to blow out the candle.

As the initial rate of burning becomes slower due to lower pressure or temperature or to the nature of the fuel, so we need greater turbulence to compensate; thus if we change from petrol to alcohol we need more turbulence and again there is little doubt but that we need more turbulence at high altitudes than at ground level, or at lower compression ratios than at high. Excessive turbulence prevents detonation and the pre-ignition which follows in its train, but in its place gives such a rapid pressure rise as to be almost equally alarming. Thanks again to the R.A.E. indicator it is possible to arrive at some idea of the relation between turbulence and spread of inflammation from the slope of the straight portion of the pressure rise. In Fig. 15 are two diagrams, taken with the R.A.E. optical

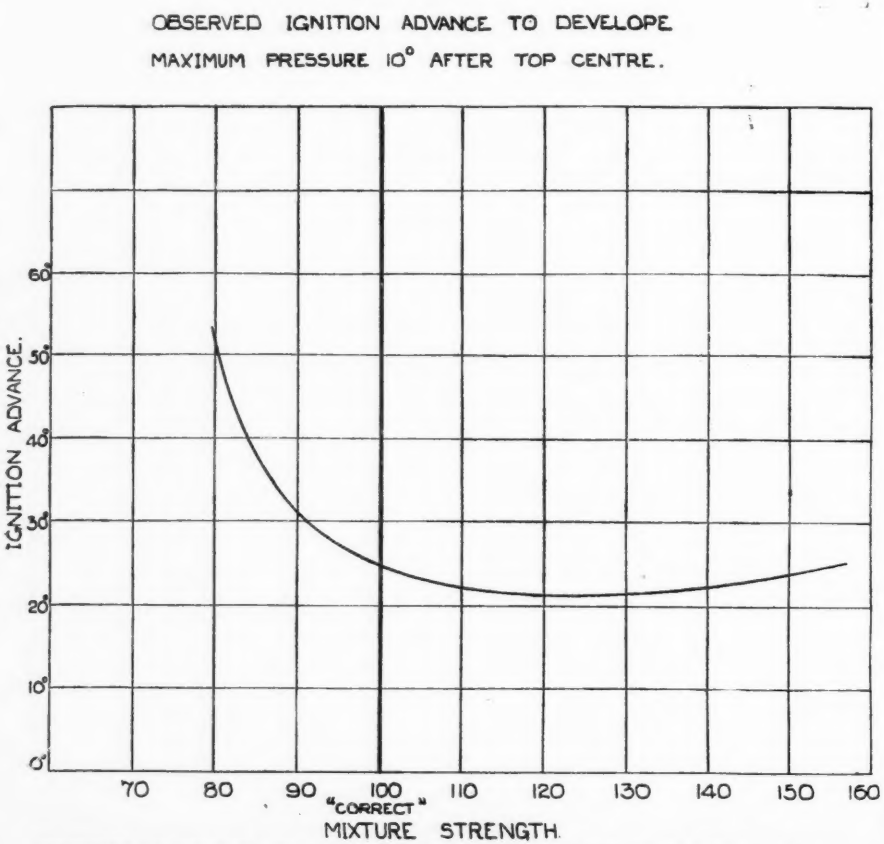


FIG. 14.

indicator from a 5½in. by 7in. cylinder with two different forms of combustion chamber—in one turbulence was due solely to the entering velocity of the mixture, in the other it was accentuated greatly just before ignition. In one case the rate of pressure rise is 20lbs. per sq in. per degree of crank-angle, in the other nearly 100lbs. per degree. In both cases exactly the same mixture strength was used and all temperatures, etc., were the same. Fig. 16 shows three diagrams taken from a variable compression engine with fixed ignition, constant mixture strength and constant speed at compression ratios of 4 : 1, 5 : 1 and 6 : 1. Here, in all cases, the turbulence must be much about the same, but there is evidently a considerable speeding up of the whole process as the compression is raised, whether due to temperature or pressure it is hard to say, for both have been changed, though the former but slightly. Fig. 17 shows, superimposed, two diagrams (*A* and *B*) taken from the same engine at the same compression ratio and under identical conditions, except that in one case the compression temperature was raised nearly 100 deg. C. as compared with the other. The more rapid burning at the higher compression temperature, particularly during the earlier stages of combustion, is very noticeable.

Influence of Temperature and Pressure of Compression

It is very important to determine once and for all how far the rate of burning and detonation are dependent upon pressure alone as distinct from temperature. While explosion vessel experiments with stagnant mixtures indicate

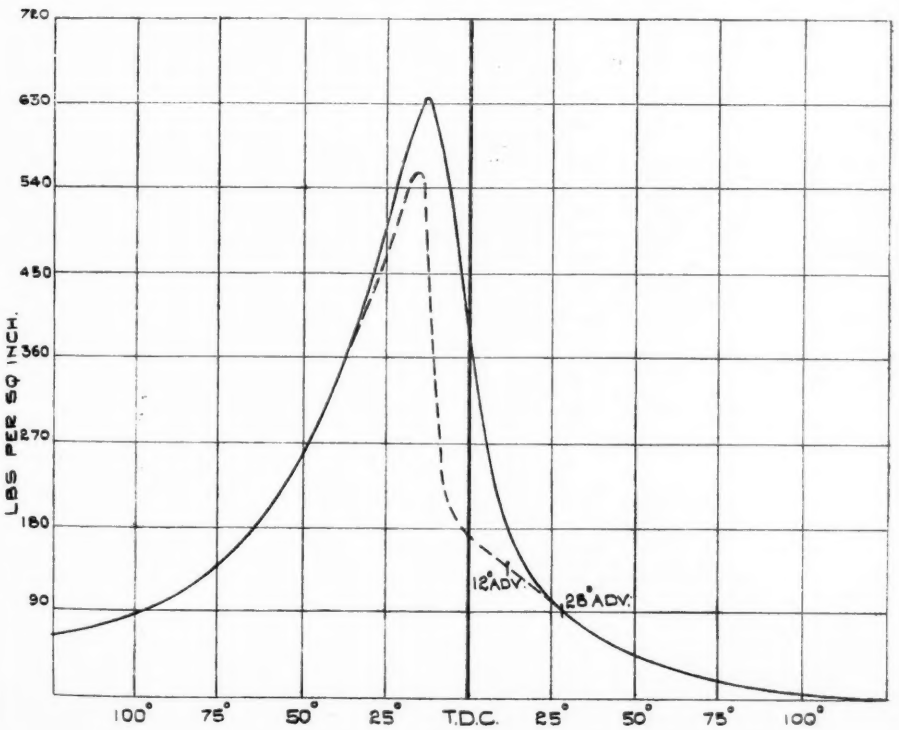
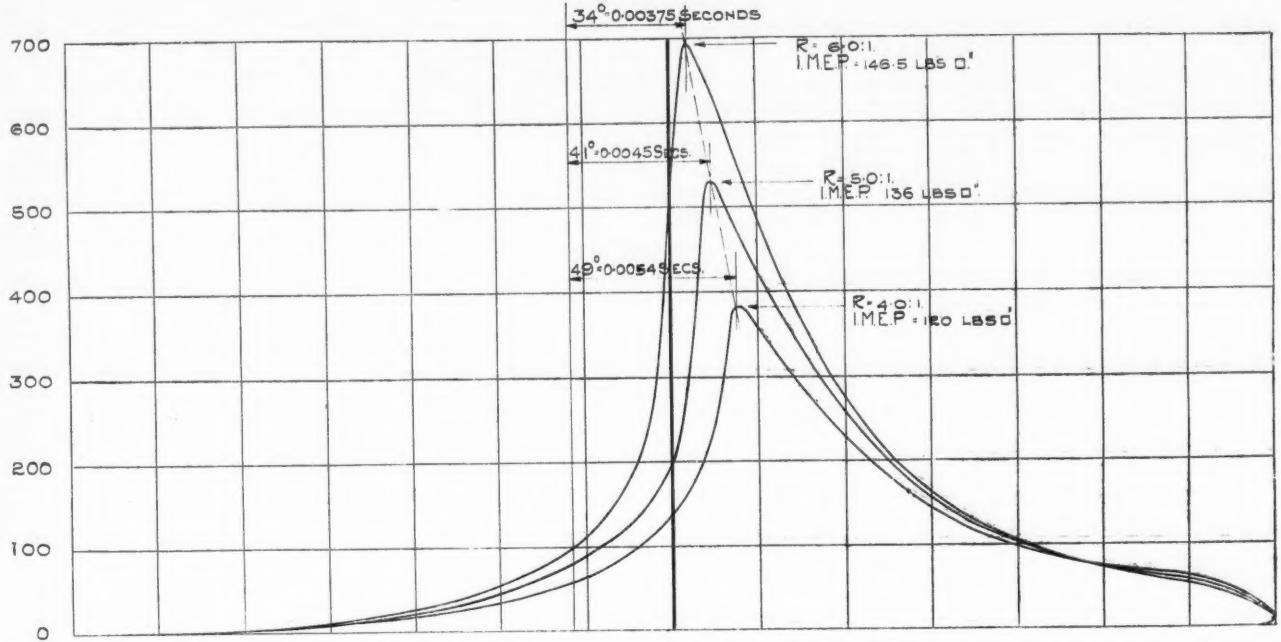
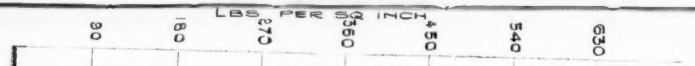


FIG. 15.



Three indicator diagrams on a time base, showing time occupied from passage of spark to maximum pressure in degrees and seconds. Taken from variable compression engine at constant mixture strength and with ignition 30° before top centre in all cases. All temperatures constant to within ±2°C. Speed constant 1,500 r.p.m.

FIG. 16.



that rate of burning is independent of pressure, yet even here the results are rather conflicting, for Fenning's tests, to which I have referred previously, show (for example in Fig. 10) a very marked increase in the tendency to detonate when the pressure was raised only slightly and the temperature kept constant, and, from the theoretical side, physicists do not appear to be in agreement. From the evidence of engine tests it would appear that pressure alone has a profound influence both on the rate of burning and on the tendency to detonate, but engine tests are not entirely satisfactory, for so many variables creep in.

As we raise the compression of an engine—

(1) We raise the pressure scale very rapidly.

(2) We raise the compression temperature very slightly indeed, when the diminished proportion of hot residual exhaust products is taken into account.

(3) By reducing the proportion of residual exhaust products which act as diluents, we raise the flame temperature very appreciably.

The writer has made many attempts to eliminate the temperature changes by controlling the initial temperature and therefore the compression temperature, and by adding additional exhaust products, as the compression is raised, in order to keep the flame temperature constant; from tests of this nature he is convinced that under actual working conditions pressure and pressure alone plays a supremely important part, but it is very difficult to prove this because the temperature control at best is somewhat indeterminate. When speaking of pressure in this connection I am not ignoring the local or indirect effects of pressure upon

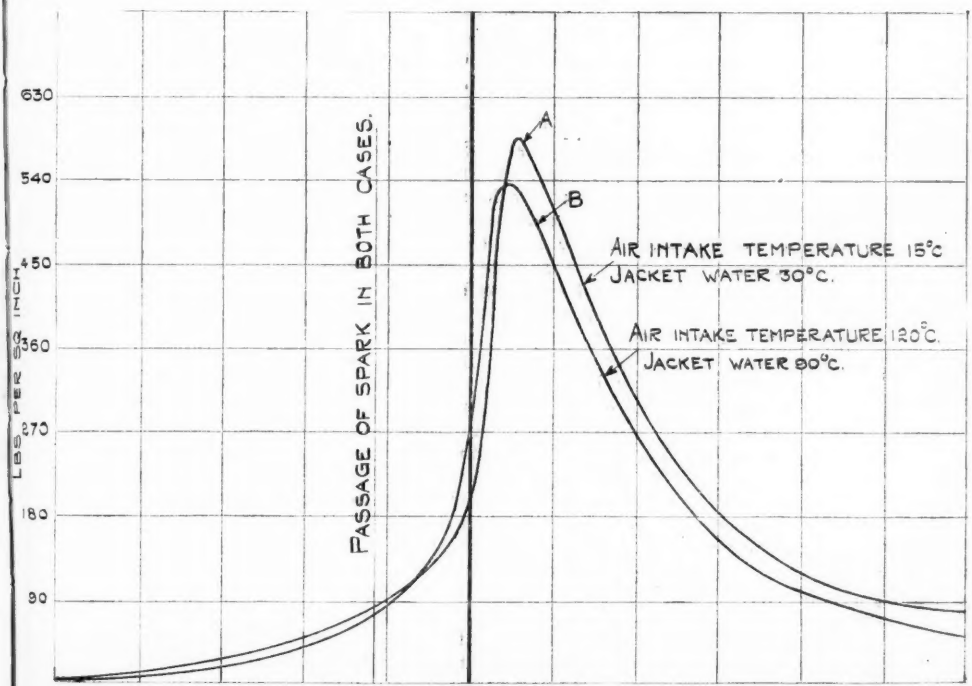


FIG. 17.

temperature, but I count these as pressure effects, since they are controlled by the pressure as distinct from the temperature of the charge. The question is of great importance in connection with aircraft engines, for, apart from the comparatively small change in atmospheric temperature (which is reduced to almost a negligible proportion when hot-water heated carburettors are used), pressure and pressure alone is the only important variable. If rate of burning and detonation are really independent of pressure, as many scientists insist, then it would seem we cannot use an appreciably higher compression ratio at altitude than on the ground. This is directly contrary to all present-day experience, but so far as the writer is aware no really accurate quantitative tests have yet been made in order to ascertain how far it is permissible or necessary to raise compression with altitude. There is every indication that it is not only permissible but even necessary, for indicator diagrams taken at high altitudes show a rapid falling off in the rate of burning, and therefore in power and efficiency over and above that due to the density alone. If we can raise compression only in proportion to the temperature variation, that is insignificant; if we can raise it anything like in proportion to the pressure variation then there is ample scope for improvement and for the use of supercharging, but we must bear in mind that if this be the case the compression injection type of engine will probably not run efficiently, if at all, at high altitudes unless its pressure is maintained by supercharging.

Reference has already been made to the value of disposing of the heat of the residual products elsewhere than into the fresh charge. The possibility of other ways of absorbing the heat left over from the previous cycle and so obtaining not only a much higher volumetric efficiency and power output but also much cooler running are well worth consideration. To inject water as a fine spray during this period proved complicated and only partially satisfactory; attempts were made to introduce it during the suction stroke, for it matters not when the heat is removed so long as its removal is accomplished before the end of the suction stroke. This also proved disappointing, for the water could not be atomised sufficiently finely to ensure its evaporation during the suction stroke, with the result that some remained in a still liquid state even at the end of the compression stroke and so absorbed some of the useful heat of combustion. As a means of checking detonation it was tolerably successful, but it proved of little value for increasing the volumetric efficiency. Later experiments showed that the desired end could be obtained and several incidental advantages secured in addition when about 5 per cent. to 10 per cent. of water was dissolved in the fuel. This had the effect of increasing the volumetric efficiency, and therefore the power output, by about 6 to 7 per cent. At the same time it cools thoroughly the exhaust valves and all internal parts. Also the lowering of the compression temperature and the presence of steam both tend to check detonation and permit of a much higher compression being used. The writer is convinced that this method of internal cooling is destined to prove of very great advantage to aero engines, particularly to air-cooled engines, for it permits of any compression ratio up to 7.0 : 1 or higher being used at ground level, while the internal cooling, the lower flame temperature, etc., all serve to keep the engine very much cooler. At the same time, owing to the greatly increased volumetric efficiency and the higher compression which can be used, the power output at ground level is increased by about 20 per cent. It is not proposed to use water in solution in the fuel under normal running conditions at altitude, but only as means of getting off the ground and attaining a height at which the engine can operate on petrol. In his previous paper before this Society the writer discussed various means of getting a high compression engine to operate at ground level; he had then to admit that all of them reduced the power output to a greater or lesser degree. The use, however, of water in solution in the fuel, appears to be by far the most hopeful method in that, unlike all the others, it gives a considerable increase in power at ground level and an equally considerable reduction in heat flow to the cylinder walls.

Since water will not dissolve in petrol or benzol some mutual solvent must be found; of such acetone appears the most satisfactory, but methyl alcohol is nearly as good and has the advantage of being more readily available in war time, or alternatively a fuel of very high latent heat may be used, such as power methylated spirit, which already contains a large percentage of water in solution. Only a relatively small quantity of the water-bearing fuel need be carried, sufficient to lift the machine perhaps three times from the ground to, say, 6,000ft., which, in the case of fast scout machines, would mean sufficient for about 15 minutes' flight. The slight extra weight of this fuel would be far more than compensated for by the much higher efficiency where running on petrol, due to the higher compression which its use renders possible.

Stratified Charge

In a previous paper which I had the honour of reading before this Society some three years ago, I referred to experiments we had made in connection with stratifying the charge, that is, separating it into two component parts, one consisting of a comparatively rich and rapid burning petrol-air mixture, in the neighbourhood of the sparking plug, and the other of pure air to act as a diluent; these two components were kept more or less separate until after ignition of the first. By such means a high initial flame temperature is obtained, but so soon as inflammation is fairly started, the flame temperature is reduced by dilution with the excess air present in the cylinder. Since that date a great deal of experimental work has been carried out along these lines and with extremely encouraging results; by such means not only is it possible to work at much lower mean flame temperatures than would otherwise be practicable, but applied to aircraft, the method has other peculiar advantages; for example, it does away with any necessity for altitude compensation, since, as the density of the air falls, and the carburettor tends to deliver a richer mixture, more and more of the excess air present in the stratified charge is consumed, with the result that the engine becomes self-compensating as regards carburation, and the power output falls directly as the flow of petrol, *i.e.*, as the square root only of the density, while at all times irregularities either of carburettor adjustment or of distribution are taken care of automatically, since, under all conditions of working, excess air is present to burn completely all the fuel. It has been found that while, with a homogeneous charge, the weakest mixture which can be used in the weakest cylinder must contain at least 82 per cent. of the fuel required for complete combustion and the average to a number of cylinders at least 90 per cent. corresponding to a flame temperature of approximately 2,360 deg. C., and an efficiency relative to the air-cycle of about 69 per cent.; with a stratified charge it is possible to work with as little as 50 per cent. of the fuel required for complete combustion, corresponding to a mean flame temperature of only about 1,800 deg. C. and a theoretical relative efficiency of somewhere about 78 per cent. Actually the flame temperature of a portion of the charge is initially much higher than 1,800 deg. C., so that one cannot expect quite so high a relative efficiency, and in fact we have not yet succeeded in doing better than 75 per cent. of the air cycle, corresponding in the experimental unit we are now running to a consumption of 0.36 lbs. of petrol per i.h.p. hour, or 0.42 lbs. per b.h.p. hour at any load between 60 per cent. and 85 per cent. maximum torque. Thus, with a stratified charge, it is possible to obtain either about 60 per cent. full load torque at an exceedingly high efficiency and low heat flow, or full torque at normal efficiency and normal heat flow, or, of course, any range between these limits, but throughout all at an efficiency exceeding the highest obtainable with a homogeneous charge, and that regardless of inequalities of distribution, etc.

Supercharging

Recently a great deal of experimental work has been carried out in con-

nection with supercharging with the object of maintaining ground level density at high altitudes Up to the present the most promising method which has been tried in flight is that of using an exhaust-driven turbo-blower. This method has several attractive features and some very severe drawbacks. Its attractive features are that it is self-governing, absorbs very little power and, thanks to exhaust back pressure, it does not disturb the balance between residual exhaust products and fresh charge, and therefore does not increase the flame temperature and produce severe detonation, as would be the case were a mechanically-driven blower to be used. It carries with it, however, the following objections :—

(1) Although the flame temperature remains nearly constant, the actual amount of heat to be disposed of increases directly as the density; thus if we double the initial pressure, we nearly double also the amount of heat flowing into the cylinder walls, exhaust valves, etc., and since it is heat flow which generally sets the limit to the power and efficiency obtainable from, and still more to the reliability of, engines, particularly air-cooled engines, it is very doubtful indeed whether any method which tends actually to increase heat flow can ever be of real practical value. Apart from this objection, the auxiliary gear involved, such as after-cooling of the compressed air and the mechanical difficulties both in connection with the turbo-blower itself and with the sealing and balancing of the carburettor induction and exhaust pipes, become very serious problems.

If, however, we work with a stratified charge we can then supercharge to almost any extent. For we start with a very much lower mean flame temperature and, even if we supercharge, to double the initial pressure, the gross heat flow to the cylinders need be little more than when running normally with a homogeneous charge, while apart from heat flow we have the following advantages :—

(1) We have only the supplementary air to supply under pressure, instead of both the main charge and the supercharge, so that a much smaller blower will suffice.

(2) We do not need to put the carburettor or any of the main pipe work under pressure, nor do we disturb its normal functioning in any way.

(3) Since only a small proportion of the total cylinder charge is compressed, after-cooling is unnecessary.

The whole problem becomes, therefore, very much simpler, but in the case of a poppet valve engine the admission of the stratified charge, whether it be at normal density or as a supercharge, involves the addition of supplementary valve gearing, which is a serious complication and prevents the system being applied to any existing aero engine. In the case of a sleeve valve engine, however, this is perfectly simple, for all that is needed are extra ports low down in the sleeve, which involve no additional complication.

I have tried to outline what I know or believe to be the chief thermo-dynamic problems in connection with aircraft or for that matter any type of petrol engines.

To sum up, our biggest item of loss is that due to change of specific heat, next comes direct heat loss to the cylinder walls, next detonation, which limits the expansion we can usefully employ, next dissociation, and next pre-burning. The first two are both directly dependent upon flame temperature, and the next two largely so. Anything therefore that we can do to reduce flame temperature will be a direct gain, but as I have endeavoured to show, with all fuels except hydrogen, we need a very high temperature to promote combustion; it would seem therefore that we must concentrate on trying to stratify the charge and by so doing try, in effect, to mislead the mixture by allowing a high flame temperature at the start but diluting it immediately afterwards.

Pre-burning is no doubt a serious trouble, for not only do we lose some of our charge, but by raising prematurely the temperature, it provokes detonation

and so forces us to use a lower compression than would otherwise be necessary. Hot exhaust valves are the main, almost the sole, cause of pre-burning. We can eliminate it almost entirely by the use of sleeve valves, which involve no highly heated surfaces in the combustion chamber; indeed the writer has found that with a sleeve valve, not only can he get a higher efficiency than with any poppet valve engine at the same compression, but he can also work at a much higher compression ratio and so secure a still further substantial gain.

Conclusion

It will be asked what is the outcome and what the practical application of all this theorising and speculation, and what after all is the ideal type of engine for aero work? The answer, I think, is that there are as many applications as there are designers and that it is still too soon to define the ideal type of aero engine. We each have our own ideals, and if the writer had at this moment to design engines for aircraft he would, for the light fast scouting machines, go for an air-cooled radial engine with poppet valves, a very high compression, using dissolved water or a fuel of very high latent heat for getting off the ground, with possibly a little, but certainly only a very little, direct supercharging at high altitudes. For the heavier classes of machine he would, in the light of present knowledge, go for a water-cooled sleeve valve engine with stratified supercharge.

I have said very little about fuels generally and their particular influence on the performance of an engine, partly because in a previous paper I dealt at some length with this phase of the subject, and partly because we have to-day, thanks to the new Air Ministry, specifications about the best fuel for aircraft that we are ever likely to get.

In conclusion, I would like to express my indebtedness to the Research Department of the Air Ministry for the magnificent and public-spirited manner in which they have financed and encouraged research work not only in connection with the more specialised problems relating to aircraft, but also research of a much wider and more fundamental nature.

Discussion

Sir Dugald Clerk, who opened the discussion, said he had listened with the utmost pleasure to his friend's, Mr. Ricardo, very able and very complete paper. He had added greatly to our knowledge of the general process of combustion in internal combustion engines, and had dealt with the special problems of the petrol engine. He could not express too strongly his admiration for Mr. Ricardo's work, as well as his admiration of the ingenuity he had applied in making apparatus for determining the ignition by compression, *i.e.*, ignition by compression only. He had made many discoveries which were of the utmost use in forwarding our knowledge of what went on in the interior of the internal combustion engine. In looking over his old work, in view of Mr. Ricardo's paper, he (Sir Dugald Clerk) had been very much struck with the close similarity in the results as between the large-cylinder gas engine and the small-cylinder petrol engine. The petrol engine, of course, presented other problems which we do not meet in internal combustion engines using gaseous fuel of any kind. Taking the relation of specific heat to gas mixture, which he had dealt with in a paper he had given at the Institution of Civil Engineers in 1907, where the specific heats of various gas engine mixtures were determined by actual compression within the cylinder, the conclusion he had come to, after discussing the whole of the arrangements and the whole of the details of that method of operating, was this—that if they took the air cycle standard, the usual formula given by Mr. Ricardo, and multiplied that by 0.8, they had got the real ideal efficiency which could be

expected from the working fluid used in this 60 h.p. engine, using coal gas and air. The figures which Mr. Ricardo and his colleagues, Messrs. Tizard and Pye, had obtained approximated very closely to that. The result came out at somewhere about 80 per cent.; Mr. Ricardo's results were about 80 per cent., and the formula he (Sir Dugald Clerk) had given at the end of his Institution of Civil Engineers' paper agreed closely with Mr. Ricardo's results for petrol engines, which was rather unexpected. In one case there was a large cylinder, 14ins. in diameter, and about 22ins. stroke, and in the other a little 3in. cylinder working with petrol, and they came out at pretty nearly the same value. With regard to dissociation, there were one or two smaller points he would like to mention—difficulties he had met with in his work. The first one was the expression used by Mr. Ricardo, namely, " rate of combustion." Like Mr. Ricardo, he had tried to differentiate, in the action occurring in the cylinder, between what Mr. Ricardo had called the " rate of combustion " and the " rate of the spread of the flame." In the old days he had called it the " rate of inflammation." He would say that the curves which had been shown, and which were obtained by Mr. Ricardo and Fenning, related to the " rate of inflammation," not the " rate of combustion," because it was the rate of the spread of the flame at a certain temperature of charge through the whole mass. Half way up, on the rising curve, it was assumed that there was a certain proportion of flame through it, and the remainder had not yet been touched at all, so that it was really a phenomenon depending on the rate of the spread of the flame and not the rate of combustion. Of course, it could be said that the rate of pressure rise was the rate of temperature rise, and so it was, but another trouble arose when they took the rate of pressure rise. Mr. Ricardo had given very high temperatures. In Professor's Coker's experiments in a small engine, using very rich coal-gas mixtures, he had had temperatures as high as 2,500°C., but the difficulty they had found was to know what was meant by a temperature of 2,500°C. For instance, in the combustion of a petrol mixture, if they knew that combustion was quite complete, then they had a standard whereby they could calculate the real temperature. He would like to know what method Mr. Ricardo had of determining when the combustion was complete. To get temperatures up to 2,700 degrees, did he assume that combustion was complete at the top?

Mr. RICARDO said he had taken the figures from Tizard and Pye.

Sir DUGALD CLERK said it seemed that some of the old puzzles still remained. However, in other respects it was rather remarkable how close the results were to the results he had obtained with the large-cylinder engine. He believed he had pointed out in 1907, if not before that, that the heat loss through the walls was much smaller than was usually accepted for several reasons, and further, that even if it were all saved, we should not get a very great increase in indicated thermal efficiency. He had looked up his paper, and the value was 7.5 per cent., whereas the figure obtained by Mr. Ricardo in his experiments with his little engine was 8 per cent. That was a most remarkable coincidence. He had expected to find a great deal of difference between the gas and the petrol engines, but it looked as though the process of combustion was pretty much the same in each case. Again, it was quite true that the air standard did not give the real value, and in his very early papers he had seen that quite clearly. All this had encouraged him to think that his own experiments were not carried out quite so crudely as he was sometimes inclined to think. He could not express too strongly his admiration of the R.A.E. indicator. Mr. Ricardo had made very many new discoveries, but, of course, in the internal combustion engine it always seemed that discoveries were rather of an invisible nature. For instance, there was turbulence, which should have been obvious thirty years before 1912, but somehow it was not. Then there was this change in volumetric efficiency. In the larger engines very often he had taken the temperature of the exhaust gases left in the

cylinder at the end by measuring the discharge with Callendar's pyrometer, and had taken that as the temperature at which the mixture was made in order to get the mean temperature before compression, but he certainly had not expected that, after the piston had travelled the whole way back and then began sucking in the charge, there was still a cooling going on. No doubt there would be some cooling, but not so much as to make a difference like that. Of course, in a small engine, with large cooling surfaces, and also very high temperatures, no doubt cooling would be more visible. He congratulated Mr. Ricardo on his very neat discovery. With regard to the influence of latent heat, there was one rather long-continued experiment which he had seen made. He had been discussing the question of the use of water in cylinders with the late Professor Bertram Hopkinson, who had built an engine at Cambridge University, or had altered it in such a way that, during compression, water was injected. He had no water jacket, but an oil jacket, and that rose to a temperature of about 130°C. During the compression he had injected a strong spray on the end of the piston, and also striking the combustion chamber cover. He had made a certain number of experiments with that method of cooling, and it had worked very well, but he wanted to try it on a larger engine. Therefore he (Sir Dugald Clerk) offered to let him have an engine to try it on at the National Gas Engine Company's engine room at Ashton-under-Lyne. It was an 18-cylinder engine, giving about 100 horse-power in the cylinder. Professor Hopkinson altered it, and set it running; he gave it an oil jacket, and it had run for three months, quite steadily, with a load which was heavier than its load would be ordinarily. He (Sir Dugald Clerk) believed Mr. Ricardo had given some figures with regard to water injection, or introducing the water into the fuel.

Mr. RICARDO said his figure was about 6 to 7 per cent.

Sir DUGALD CLERK said that Professor Hopkinson and himself had obtained, roughly, an increase in actual power of something over 8 per cent. in this big cylinder, but they had a diminution in thermal efficiency of 5 per cent. Unfortunately, the injecting apparatus gave out after three months. It had got red hot, and there was no water round it, and in the few minutes which elapsed before the man in charge had noticed it, the combustion chamber cracked. It was very interesting, however, and he believed the method of injecting the water at the end of the stroke could be used with advantage. With regard to the values for dissociation, in the British Association Gaseous Explosions Committee they had come to the conclusion that they could not detect dissociation at all up to 2,000°C. Mr. Ricardo had got up to 1,500°C., and had detected dissociation after that, and he would like to know how that dissociation was detected. It was very difficult to make experiments with dissociation. He had not been able to analyse it to the extent mentioned by Mr. Ricardo, but Professor David and he had been analysing it in another way. He had never been able to separate dissociation effect completely from continuity of combustion. It would be remembered that Professor Hopkinson, in former days, used to think that continued combustion was impossible—combustion going on at maximum temperature—but when they took Fenning's results, one of the principal ones was that when he heated up the cylinder to about 300 degrees he might put in a mixture of air and petrol as quickly as he liked, and the combustion seemed to begin at 300 degrees without any flame. It was known that the hydrogen burned at 300 degrees, that if they heated up the whole mass to 300 degrees there was a slow combustion going on, and actually chemical combination of carbon and hydrogen in petrol to form carbonic acid, and yet there was no flame production. Of course, if they thought that over, they got slow combustion under those conditions. In conclusion, he congratulated the Society on having such a very excellent paper presented to it, and Mr. Ricardo on his ingenuity and his ability in following up very difficult reactions of flame combustion.

Commander F. L. M. BOOTHBY, referring to hydrogen, pointed out that Mr. Ricardo had said that a small whiff of hydrogen was sufficient in a petrol engine to ignite a charge of a very weak mixture. He knew that engines ran on very little, but had never found out exactly what a "small whiff" amounted to.

Mr. RICARDO said about 8 per cent. of the weight of the fuel would operate and give a fuel mixture with half-complete combustion. It was 4 per cent. of the weight of the fuel required for complete combustion.

Commander BOOTHBY said that showed what a very small actual volume of hydrogen was required.

Mr. RICARDO replied that that was by weight.

Commander BOOTHBY said he wanted to know the volume, as it was easier to use as a basis of calculation in practice.

Mr. RICARDO said it was roughly one-fortieth of the volume of the air.

Commander BOOTHBY said he had in mind that it was possible to get 1,000 cub. ft. of hydrogen from 40 lbs. of hydrolith (lithium hydnide) and 60 lbs. of water. Hydrolith could be obtained from France, where it was used in the kite balloon service. There seemed to be a possibility, on an aero engine, of being able to generate hydrogen fuel in the same way as an ordinary acetylene generator made gas, and he wanted to know whether work had been done on that problem with a view to producing a heavy fuel aeroplane engine.

Mr. RICARDO said he did not know; so far, he and his colleagues had considered it rather outside their authority.

Major WIMPERIS said that Mr. Ricardo had answered most nobly the challenge thrown out to him to produce a paper on the thermo-dynamics of aircraft engines, and he was sure that those present were delighted with the way in which it had been done. He had one or two questions to ask, but before he came to them he would like to say that, immediately he had read the first few opening lines of the paper, he was filled with contrition, because of the remark "the pure dry air so beloved of the text book." He had not dared to look at any of his own text books; he had, however, glanced at one book on his shelves to see if he could find the guilty phrase, and he did. The author's name was H. R. Ricardo. (Laughter.) Coming to his questions, the author had said, when speaking of dissociation, "Fortunately it does not play such havoc with the power and efficiency as change of specific heat." The Gaseous Explosions Committee, to which reference had been made, said Major Wimperis, did not distinguish between those two in the curves it had shown. It had shown curves of "apparent" specific heats which grouped these two things together. Of course, it was convenient to divide them, and work had been done which enabled that division to be made, but he did not quite see how the effects of the two were as different as Mr. Ricardo had suggested in this part of the paper. Therefore he (the speaker) would be glad if Mr. Ricardo would expand on that particular point. Another point was in connection with detonation. He supposed that nobody really knew accurately what happened in a detonating explosion, what caused detonation, what was the mechanism of detonation, but he certainly did not think it was as suggested in the paper, in which Mr. Ricardo said that ". . . if at any period the rate of propagation exceeded a certain limiting figure, depending on the nature of the fuel, a detonation wave would be set up." He would like very much to know what authority there was for that statement, because it seemed to him a little unexpected. He rather pictured detonation as arising when the wave motion, inevitably started during ignition, was such as to have a temperature in the wave front which was up to the ignition temperature of the gaseous mixture. That, of course, was only surmise. There was another question. Would Mr. Ricardo kindly explain exactly what he meant when he said that the compression-injection type of engine would not run at all at high temperatures unless pressure was

obtained by supercharging? Another question arose later on, when the author said that severe detonation would be produced were a mechanically-driven blower to be used. There are no detailed reasons given for that statement, and he was sure that it would be interesting to have them. Finally, at the end of the paper the author had referred to the ideal fuel, according to a certain specification. When he expressed the opinion he did, was he bearing in mind the work which had been done in the United States in producing what was called ethyl gas, and was being sold at a few cents per gallon above the ordinary price? Had Mr. Ricardo considered such fuels when he had made the remark referred to?

Wing-Commander T. R. CAVE-BROWNE-CAVE said Mr. Ricardo had referred to the turbulence with a sleeve valve engine as being greater than that produced by poppet valves. He was not quite clear what form the turbulence took—whether the sleeve gave a greater general swirl as distinct from the smaller eddies made by the poppet valve. Could Mr. Ricardo say what was the speed of the mixture past the ignition point? If that figure could be given, even within 100 per cent., it would be of interest for certain other work.

The author had referred to the " boost " which could be given to an engine for getting the aircraft off the ground. That was of exceptional value, certainly for seaplanes, where the machines could fly easily with any load with which they could get off, and, he believed, in a less degree for aeroplanes. He did want to draw attention to the great importance of this possibility which could be achieved in existing carburettors with nothing but an adjustment due to the variable jet.

Mr. RICARDO: And a second float chamber.

Wing-Commander CAVE-BROWNE-CAVE agreed. Continuing, he said he was very much interested in what the author had said about the factors which caused overheating. He had summarised them as burnt exhaust valves, burned sparking plugs, and over-heated pistons. He (the speaker) imagined that the stratified charge would get over the difficulty of the over-heated pistons, and the consequent troubles of carbonised oil, etc. That left designers free to concentrate on the other causes of trouble. Looking at the water circulation, in all the aero engines of which he remembered details, he thought that if circulation were concentrated on those vital points only, other circulation being incidental, much improvement was possible. The vital points were the seatings and guides of the exhaust valves, and to a considerably less extent the sparking plug (because he believed it was the *central* electrode of the sparking plug that gave the trouble). The author had suggested that the cure for this was the sleeve valve. He (the speaker) entirely agreed and would have liked more details of what had been achieved with the sleeve valve engine. The author had made a very strong case for the sleeve valve and for stratified charge.

As to the point raised by Commander Boothby, he considered it was significant that the quantity of hydrogen which gave these very remarkable fuel efficiencies was considerably less than the quantity of hydrogen which became available in an airship when they had burned a corresponding amount of petrol. Wing-Commander Cave joined with other speakers in congratulating Mr. Ricardo on his paper, and pointed out that this appeared to be one of the branches of aeronautics in which research was well ahead of practical application. That was perhaps due to the extraordinary genius of Mr. Ricardo, and to the fact that he had provided himself, at Shoreham, with what might be described as such a wonderful box of tools.

Colonel TIZARD said that with regard to the thermal efficiency of internal combustion engines, he thought that although Mr. Ricardo's views were correct, the way he expressed them was wrong in certain particulars. For instance, he had said that the biggest item of loss to be dealt with was that due to high specific heat, but it seemed to him (Colonel Tizard) that this was only called a loss because we compared the efficiency of an engine with that of a perfectly imaginary engine

working on the " ideal " air cycle. It was not a real loss at all. If Mr. Ricardo were provided with a working fluid which had the properties of air, he could not make a satisfactory engine; indeed, if the working fluid had the specific heat of air, the internal combustion engine would probably never have existed. It was unkind to abuse the actual working fluid for having those properties which made the internal combustion engine an attainable reality. He objected, therefore, on principle to the statement that the biggest " loss " was that due to specific heat. If it were not for heat losses, and for limitations in material which prevented very high pressures and temperatures being employed, it would be possible to design an engine which would give the highest efficiency in spite of high specific heat and dissociation. For suppose one compressed carbon monoxide and air suddenly and then mixed them at so high a temperature that there was complete dissociation, and then expanded slowly so that gradual combination took place all the way down the expansion stroke until the original temperature was reached, then if there were no heat losses, such a process would give the maximum amount of work possible, which would be very nearly equal to the heat of combustion of carbon monoxide.

All this might appear rather academic, and he did not wish to imply that the imaginary " air cycle " was not of great practical assistance, but only that departures from " air cycle " efficiencies should not be considered as avoidable " losses."

Continuing, he agreed with Sir Dugald Clerk that Mr. Ricardo had put forward a plausible explanation of decrease in volumetric efficiency with increase in compression ratio, but he thought that it would have to be worked out in greater detail before it could be accepted as the real explanation. Sir Dugald Clerk had referred to the difficulty of differentiating between so-called dissociation and delayed combustion. He (Colonel Tizard) considered that the question to be considered in this connection was as follows:—Is there thermo-dynamic equilibrium at the maximum temperature? If, so, one should not talk about delayed combustion, since this implied that the uncombined carbon monoxide and oxygen would combine if left long enough at the same high temperature. If equilibrium were attained, they would not combine however long they were left. The results of Dr. Ricardo's measurements of thermal efficiencies were so close to those which could be calculated theoretically on the assumption that thermo-dynamic equilibrium was attained at the maximum explosion temperature that he thought that there was very strong indirect proof that in a modern petrol engine at any rate there was very little " delayed combustion," though there might be considerable dissociation.

Mr. RICARDO, replying to the discussion, thanked Sir Dugald Clerk for his kind remarks. One of the most interesting points which Sir Dugald Clerk had raised was how closely a small high-speed petrol engine appeared to agree with the performance of a large and slow-running gas engine. It appeared to be a combination of circumstances, which apparently cancelled out, because there were many very fundamental differences between the two. With regard to the question of combustion and inflammation, he was afraid that he ought to have used the word " inflammation " in that connection and not " combustion." As to heat loss, Sir Dugald Clerk had mentioned that he had found a $7\frac{1}{2}$ per cent. effective heat loss, whereas the figure which he (Mr. Ricardo) had taken was 8 per cent., in a petrol engine cylinder of $4\frac{1}{2}$ins. diameter, running at a piston speed of 2,000ft. per minute. The piston speed no doubt partly balanced the difference in size, and he took it that that was practically three times that of the larger gas engine. He was particularly interested in Sir Dugald Clerk's figures with regard to Professor Hopkinson's test with water injection, showing an increase of 8 per cent. in power, and therefore still higher in volumetric efficiency, since the thermal efficiency was down. Where the water was introduced with the fuel into the

cylinder, he and his colleagues had reason to believe that it was almost entirely evaporated. It was rather a different case, but they had also found a loss of efficiency of the same sort of order. With regard to cooling residual exhaust products at the beginning of the suction stroke, they had estimated that their products had a temperature of the order of 1,000°C. at the completion of the exhaust stroke.

Sir DUGALD CLERK said the temperatures, he believed, were very much lower —about 700°Fah.

Mr. RICARDO referred to Commander Boothby's inquiry as to the quantity of hydrogen required. He (Mr. Ricardo) had given the figure roughly as about one-fortieth by volume of the air taken into the engine. He would be very interested to hear what sort of gross weight the hydrogen plant involved, and how it compared with the weight of the hydrogen yielded.

Commander BOOTHBY said it was about 100 lbs. of material for 1,000 cub. ft.

Mr. RICARDO replied, speaking off-hand, that he was afraid that would put it out of court; he did not think it would be a practical proposition at that weight.

With regard to Major Wimperis's remarks, Mr. Ricardo had to admit that it was probably inaccurate to distinguish between change of specific heat and dissociation in so far as their influence on thermal efficiency was concerned, and he was grateful to Major Wimperis for calling attention to this. With regard to detonation, he agreed that as yet we had insufficient knowledge to dogmatise about this, but he thought he had covered himself by his opening remarks, in which he stated that many of the arguments he would use were little more than mere speculation. He, however, still preferred his own diagnosis to that of Major Wimperis, though he had quite an open mind on the subject. Major Wimperis had also asked him to enlarge on the question as to why the compression-ignition type of engine must involve supercharging. The assumption there was that we depend on pressure for the necessary rate of inflammation. We wanted the maximum possible rate of inflammation in this type of engine, and it might be that we should have to maintain the pressure by supercharging. Major Wimperis had also asked him to enlarge upon the question as to why severe detonation was produced with the mechanically-driven blower as compared with the exhaust-driven one. What he had had in mind was that with the exhaust-driven blower there was an exhaust back pressure which was at least equal to the ordinary suction pressure. If a mechanically-driven blower were used it would disturb the balance since the proportion of residual exhaust products would be reduced; this would give the same effect as scavenging, and any attempt at scavenging produced the most violent detonation, because it raised the flame temperature. Major Wimperis had also asked whether he (Mr. Ricardo), when referring to the Air Ministry specifications, had had in mind tetra-ethyl-lead. He and his colleagues had made a number of experiments on that, but he did not think it had been extremely satisfactory. They had made one continuous run of fifteen hours; after about ten hours the engine began to labour heavily, and after fifteen hours it had seized up entirely. They had then dismantled it, and had found that the piston had seized, and the whole of the inside of the cylinder looked as if it had been heavily distempered. The valves were heavily coated, and the engine was completely stuck up and had to be entirely dismantled and thoroughly cleaned out, so that until there was some improvement he did not think tetra-ethyl-lead would do.

Major WIMPERIS said that the Americans added a little of something else, but did not say what it was.

Mr. RICARDO said it was carbon tetrachloride. He and his colleagues were trying it, but so far it did not appear to help matters very much.

Referring to Commander Cave-Browne-Cave's question as to the velocity of the air past the plug in the engine, Mr. Ricardo said he could not attempt to give any figure at all, and he saw no means of measuring it. He would not like even to guess it. As regards the difference in turbulence between the sleeve valve engine and the poppet valve engine, he believed it was partly due to the fact that the gas entered as a solid stream instead of as a hollow cone, the orifice coefficient of the sleeve valve was very much higher, and also that in the sleeve valve he had been using it entered more or less tangentially and apparently kept up the swirl. Certainly in that type of engine the tendency was to cause excessive turbulence, and they had had to use a conical head to try and kill the turbulence, but even so it appeared to be greater than in a poppet valve engine. With regard to the cooling of the exhaust valves, he heartily agreed that everything possible should be done to improve that, but he thought that the factor which was really important was the latent heat of the water. So long as the exhaust valve was thoroughly immersed in water, and so long as there was plenty of room for the steam to get away, he was not at all convinced that a very rapid circulation was of any particular value. He had tried experiments using a very high boiling point liquid as a cooling agent, and that was disastrous; he had to keep a very high velocity circulation over every point where there was a high heat flow, but with water he doubted whether it mattered, because one had such a large reserve capacity in the form of latent heat. As to figures for the sleeve valve engines, he was running experimentally a large single-cylinder sleeve valve engine, and the best results up to date were a brake mean pressure of 140 lbs. per sq. in. at from 1,200 to 1,400 revolutions. The mechanical efficiency was 89 per cent., so that the indicated mean pressure was 158; the fuel consumption was 0.43 lbs. of aviation spirit per brake horse-power hour, and the oil consumption 0.015 lbs. Those were quite good results.

With regard to Colonel Tizard's remarks, Mr. Ricardo quite agreed with Colonel Tizard that the specific heat did not represent a loss in the true sense of the word—discrepancy would have been a better word. Strictly speaking, no doubt the efficiency of any internal combustion engine should be compared with the ideal efficiency possible in the circumstances under which it was operating; the trouble is that in this case the ideal changes with every change of circumstance, even to changes of mixture strength, cylinder temperature, etc. In spite of Colonel Tizard's remarks, he still felt convinced that the air cycle remained the only practical basis for comparison, because though imaginary and unattainable, it is at least a constant quantity, and not a fickle will o' the wisp like the true ideal. With regard to the interchange of heat between the residual exhaust products and the cylinder walls, while he quite agreed with Sir Dugald Clerk and Mr. Tizard that this cannot be of a large order, yet consideration would show that only a very small loss would serve to account for the observed change in volumetric efficiency as between different compression ratios.

On the motion of the CHAIRMAN, a hearty vote of thanks was accorded Mr. Ricardo for his excellent paper, and also to those who had taken part in the discussion.

This concluded the proceedings.

Colonel THE MASTER OF SEMPILL (*contributed*): I have read Mr. Ricardo's very remarkable paper with great appreciation, and think the conclusions arrived at, coming from such an authority, are of supreme interest.

Everyone will agree that perhaps the most important factor in the development of aviation is the production of engines of greater reliability. Mr. Ricardo points out that for the heavier class of machine he would develop the "sleeve valve" engine. I feel that such a conclusion will be a surprise to many, and would like to know which of the two known types he would give preference to in

this connection. There can be little doubt that an engine of this type would be more reliable; some of the difficulties which occur with the " poppet valve " type of engine, about which we have heard to-night, would disappear.

Has Mr. Ricardo considered the Fullagar engine which has certain of the features stressed by him as being desirable in a modern aero engine?

I think the Society owes a very great deal to Mr. Ricardo for his contribution of this remarkably interesting paper.

Flying Officer C. J. Sims, D.F.C., R.A.F. (*communicated*): The paper of Mr. H. Ricardo is of great value in that it is a résumé of what is at present known of the thermo-dynamics of high speed and high performance internal combustion engines. Its value is further increased by providing a large amount of material for further speculation. Certain parts of the paper require amplification, as they were dealt with very briefly and are apt to be rather confusing.

Specific Heat

The efficiency of an engine is given by the equation $E = 1 - (1/r)^{\gamma-1}$ where r is the ratio of expansion and γ is usually taken as 1.41. This value of γ is the correct value for an external combustion engine, but not for an internal combustion engine. When combustion takes place steam is formed and the specific heat of gas is raised, the formation of steam depending on the mixture strength and the fuel. No longer is air the working fluid.

The minimum temperature at which steam can exist at atmospheric pressure is 100°C. Therefore for each mixture strength and each fuel γ should be obtained at 100°C., this value giving the maximum efficiency obtainable in an internal combustion engine. Any increase of the specific heat from this value will be a direct loss of efficiency, which can be varied by varying the maximum temperature.

The increase of specific heat with temperature rise leads to a loss of efficiency in two directions—when the gas is at maximum temperature extra heat is absorbed by the increase of specific heat, but as external work is done the temperature will fall and the specific heat will fall. Some of the extra heat absorbed will now be given out, but as this is not added at constant volume there is a loss of efficiency. Secondly the gas will be at a temperature of about 1,000°C. when the exhaust valve opens; the increase of specific heat up to this value means a dead loss.

With a slow-speed engine there will be greater cooling than with a high-speed engine, the temperatures will be less and the loss due to increase of specific heat less. It still remains to be determined what is the optimum speed at which to run in order to reduce these combined losses to a minimum.

The loss due to change of specific heat will be much less with a constant pressure compression-ignition engine than in a constant volume engine with its much lower compression ratio. The maximum temperature is lower, the expansion ratio greater, and therefore the exhaust temperature lower.

Dissociation

At high temperatures H_2O and CO_2 become dissociated. It is usually assumed that when external work is done, and the temperature falls, there will be a recombination and the heat absorbed at the higher temperatures will now be given out, but with a loss of efficiency as the heat is not added at constant volume. This is not necessarily so. If the gas be in a dissociated state and suddenly cooled, there will be no reassociation. This cooling method has been satisfactorily used to determine the amount of dissociation.

An engine with a high piston speed would expand the dissociated gases at such a rate that complete reassociation would not take place, thus giving a direct

loss of efficiency. The slower the speed of the engine the greater the reassociation, but the greater external cooling loss. The optimum speed in order to obtain the best results remains to be found.

Again the constant pressure compression-ignition engine scores heavily. Dissociation will be less due to both decrease in maximum temperature and increase in pressure.

Decrease in Volumetric Efficiency with Increased Compression Ratio

When a fresh charge is taken into an engine it is heated partly by the engine and partly by the residual exhaust gases. So long as the heat is taken from the exhaust gases there will be no change in volumetric efficiency, as the expansion of the fresh charge is compensated by the contraction of the exhaust gases. But all the heat taken from the engine leads to a direct loss of volumetric efficiency.

When the compression ratio of an engine is increased, the compression and explosion temperatures increase and the mean temperature of the engine rises. More heat is taken from the engine and less from the exhaust products by the fresh charge and the volumetric efficiency falls.

Mr. Ricardo's theory would give a Diesel type engine a low volumetric efficiency, for this type of engine has a high compression ratio and low exhaust temperature. After making due allowance for the slower speed of the Diesel, larger values, etc., the volumetric efficiency found in practice is too great to allow Mr. Ricardo's theory to be tenable. This engine runs on a low temperature cycle and the mean temperature of the engine is low, with a resultant high volumetric efficiency.

If a high volumetric efficiency is to be obtained on a high compression ratio engine of the constant volume type, then as the compression ratio increases, the cooling will have to be more efficient, and there is no need for the suggested valve arrangements.

By injecting water into the cylinder Mr. Ricardo obtained greater power and larger volumetric efficiency, again by increased cooling of the engine. But fighting aircraft fitted with internal water-cooling devices are more vulnerable than those without. Failure of such a device, whether by mechanical breakdown or enemy action, would mean that the power would decrease to a serious extent, placing the pilot of that machine in a precarious position. The compression of an internally water-cooled engine would be such that if the water supply failed, the engine would detonate, for such a compression ratio would balance the loss of thermal efficiency. Thus the cessation of the water supply would give a loss of power due to decrease of volumetric efficiency, and that due to the necessity of retarding the spark or throttling down.

Increased cooling could be obtained by a greater use of light alloys. Electron is such an alloy which could be used to great advantage for pistons. A lithium aluminium magnesium alloy, if it could be obtained economically, would be of great value as the specific heat of lithium is equal to that of water.

Better external cooling would be of greater value than internal cooling, but the lower temperature of the Diesel type engine is attractive.

Latent Heat

The total calorific value of a liquid fuel is absorbed in an engine by :—

1. Latent heat of evaporation of the fuel.
2. Latent heat of steam.
3. Work done.
4. Waste or lost heat.

The fuel has to supply the heat for 1 and 2, and this cannot be avoided.

Determinations of the calorific value of a fuel usually give the calorific value minus 1 and 2. This is the correct value to take.

In an engine the fuel always supplies the heat from its own combustion; if it obtains this heat from waste heat so much the better, but this waste heat came from combustion. If a correction is to be applied when the fuel takes its heat from waste heat, then also a correction must be applied for all the other heat which the gases absorb during the suction and beginning of compression strokes. This would mean that it would be an impossibility to find the thermal efficiency.

An injection engine will have a smaller thermal efficiency than a carburettor engine, because in the case of the injection engine the heat is absorbed from the working strokes, but this automatically decreases the work done, thus giving a lower thermal efficiency, which is work done divided by the lower calorific value of the fuel. It is incorrect to add the latent heat of the fuel to the calorific value, for now the work done will be smaller and the calorific value greater; the allowance for the latent heat being made twice will give too small a thermal efficiency.

Mr. W. J. STERN (*contributed*): Mr. Ricardo's most excellent paper touches on most points affecting the future development of aero engines.

Mr. Ricardo makes it clear that several of his conclusions are based on personal opinion rather than on experiment.

In such instances alternative explanations appear possible. A case in point is the effect of altitude on compression ratio. The possibility of increasing the compression ratio at altitude is stated by Mr. Ricardo to prove that detonation depends primarily on compression pressures, since variations in the compression temperature with altitude are negligible. This would mean that the suction temperature is practically independent of altitude, or in other words of the engine load. Whilst the suction temperature will not be much affected by variation of the outside air temperature, it will certainly be affected by the engine load, and the result may be a considerable decrease of the compression temperature with altitude.

The observed facts can thus be equally well explained by the supposition that compression temperature rather than pressure is the controlling factor of detonation, a view which on theoretical grounds has much to recommend itself.

Mr. RICARDO's reply to Colonel The Master of Sempill and Mr. Sims:—

I should like first to thank Colonel The Master of Sempill most sincerely for his very flattering and encouraging remarks.

With regard to the type of sleeve valve, nearly all the experience I have had so far has been gained with the Burt single-sleeve type; with this I have had excellent results, that is to say, a somewhat higher mean pressure at medium piston speeds, *i.e.*, 1,200 to 1,800 feet per minute, and a substantially better fuel economy at all speeds than I have yet been able to obtain from any type of poppet valve engine. So far, my experience with double reciprocating sleeves has not been nearly so satisfactory, chiefly owing to lubrication troubles.

With regard to the Fullagar engine, I feel doubtful whether any engine operating on the two-stroke cycle is likely to prove practicable for aircraft, unless or until the compression-igaition cycle is further developed, otherwise I fear that the loss of unburnt charge through the exhaust ports, which is almost unavoidable in the case of the ordinary cycle, will render the engine too extravagant in fuel. Also, the general shape of the Fullagar engine, though suitable for marine or stationary work, is not at all well adapted to aircraft. I quite agree that this engine contains features which are, in my opinion, most valuable, but they would, I think, be of more value still if they could be applied to a four-stroke cycle engine.

I have read with the greatest interest Mr. Sims' most pertinent remarks. It

is difficult in the scope of a paper to elaborate one's statements as much as one would like to be able to do, and as a result one must necessarily omit many qualifying conditions.

First, with regard to specific heat.

(*a*) I quite agree that the air-cycle formula is applicable only as a basis of comparison, as indeed I stated, but I do not see the object of trying to amend it to allow for the specific heat of steam formed. Steam is not the only product of combustion, and if the simple air-cycle formula is to be departed from, surely the only reasonable alternative is to calculate out for the specific heat of the actual products of combustion which will vary with every change in mixture, strength, etc.? I can see no merit in any intermediate stage which has all the complications of the true ideal formula without bringing us much nearer to the actual ideal value.

I agree that there is probably an optimum speed, though so many factors enter in that the balance between specific heat and direct heat loss becomes only a very secondary issue.

I agree also that the loss due to specific heat becomes less in the case of the compression-injection engine, because of the lower mean flame temperature and the longer expansion, but I thought I had already made this point quite clear.

Dissociation

Mr. Sims' point that the degree of reassociation will depend in some measure on the time element and therefore on the speed of rotation is an extremely interesting one, and I am most grateful to him for bringing it forward.

With regard to the compression-ignition engine, I am doubtful whether this really scores very much. Although the mean flame temperature is lower (though only a very little lower on full load), the local flame temperature is very nearly as high.

Again, the maximum pressure in a compression-ignition engine is not, or should not be, any higher than in an explosion engine if the former is to be of a weight applicable to aircraft. In actual practice the maximum pressure at ground level is about 600-650 lbs. per square inch in an ordinary explosion cycle aero engine with a compression ratio of 5.8 : 1, and it is the proud boast of the compression-ignition engine that this maximum pressure need not be exceeded.

Volumetric Efficiency

The statement in the second paragraph of Mr. Sims' contribution, namely, that the mean temperature of the cylinder rises with increase of compression, is directly contrary to all theory and practice; for, despite the very small increase in compression temperature, the mean cycle temperature is very much lower. Provided there is no detonation the mean temperature of the cylinder naturally falls with increase of compression ratio, a fact which can be demonstrated very easily with a variable compression engine.

I shall be very interested to learn of any accurate tests on the volumetric efficiency of a Diesel engine. I can find none which are in any way comparable with those taken from explosion cycle engines, though I hope very shortly to be able to make some comparative measurements on an engine which will operate at will, either as a Diesel or an explosion engine, without any change beyond that of varying compression. Meanwhile, all the available evidence I have been able to find tends to confirm that the volumetric efficiency of a Diesel engine is actually somewhat lower than that of a low compression explosion engine.

With regard to injecting water for internal cooling, I stated quite clearly that we had found this most unsatisfactory. The only way in which we could

obtain the desired effect was by dissolving the water in the fuel—a method which avoids the objections raised by Mr. Sims.

I do not think we have any evidence to show that any appreciable gain in volumetric efficiency can be obtained by the use of light alloy pistons, nor would such be expected seeing how small and remote a part of the total heating surface the piston head forms during, say, the latter part of the suction stroke. On the other hand there is a mass of cumulative evidence by many independent experimenters to show that the temperature of the piston crown has only a very insignificant influence on the volumetric efficiency. Better external cooling would, of course, always be desirable, but can that be more than a pious hope? We all do what we can to get as much external cooling as possible.

Latent Heat

I am afraid that I cannot quite follow the arguments used here. Surely it must be clear that in the case of a Diesel engine the latent heat of evaporation of the fuel must be supplied from the *useful* heat of combustion of the fuel, and therefore represents a loss, while in an explosion engine it is supplied from purely waste heat. Of course, in either case the heat is obtained originally from the heat of combustion of the fuel, but in the one case before it has done its work, in the other case afterwards, two very different conditions, and it was my object to emphasise this difference.

R.38 MEMORIAL PRIZE, 1923

The Council have decided, for this year only, to increase the amount of the prize from 25 guineas to 40 guineas and to divide this sum between the papers on "The Aerodynamical Characteristics of the Airship as Deduced from Experiments on Models, with Application to Motion in a Horizontal Plane," by Mr. R. Jones, M.A., and "A Detailed Consideration of the Effect of Meteorological Conditions on Airships," by Lieut.-Col. V. C. Richmond, O.B.E., A.F.R.Ae.S., and Major G. H. Scott, C.B.E., A.F.C. They also desire specially to commend the paper on "The Strength of Rigid Airships," by Mr. C. P. Burgess, Commander J. C. Hunsaker, U.S.N., Hon.F.R.Ae.S., and Mr. Starr Truscott. All three papers will be published in the Journal.

R.38 MEMORIAL PRIZE PAPER.

THE AERODYNAMICAL CHARACTERISTICS OF THE AIRSHIP AS DEDUCED FROM EXPERIMENTS ON MODELS, WITH APPLICATION TO MOTION IN A HORIZONTAL PLANE

BY R. JONES, M.A.

I. *Introduction*

1.0. When confronted with the problem of selecting a subject suitable for an essay to present for the "R.38 Memorial Prize" of the Royal Aeronautical Society, several considerations indicated the desirability of dealing with experiments on airship models and their application to full-scale. The analyses of the results of such experiments, with applications, have from time to time been issued by H.M. Stationery Office as reports of the Aeronautical Research Committee. As far as is known, however, no comprehensive survey of the aerodynamics of the airship with a concise summary of the experimental data available has yet been published. This does not apply to the same extent to the aeroplane; it is but natural that, bearing in mind the hitherto more extensive use of the heavier-than-air machine, more attention has been given to it in technical papers and aeronautical text books than has been directed to the airship. In the case of the aeroplane more data have been collected and facilities for full-scale research have been more plentiful, so that the gap between model and full-scale has been more adequately bridged. Until comparatively recent years, full-scale experiments upon airships were practically unknown in this country. Fortunately, however, owing to the untiring zeal and energy of those pioneers of full-scale airship research, some of whom lost their lives in the disaster to R.38, observations of the motion of an actual airship in flight have been obtained, and the availability of such observations has been of enormous assistance in showing how the results of experiments on models can be applied most advantageously to predict full-scale performance.

The prediction of the general performance and stability of an airship from model data is in itself a matter of great importance for future design. Experience appears to indicate another point of possibly greater importance to be considered by the designer, viz., a systematic examination of the strength of the structure of the airship.* It is probable that the aerodynamical data having the most important

* The Airship Stressing Panel of the Aeronautical Research Committee has already made considerable progress with this work. R. & M. 800.

bearing on this question are those dealing with the distribution of pressure over the airship under various conditions of motion. It is obvious that before such data can be adequately applied, methods of calculating the motion of the airship under different circumstances must be available, and we deduce that the prediction of general performance is of fundamental interest from the point of view of strength calculations. The conclusions of the expert committee who examined the cause of the R.38 disaster clearly confirm this. The results of model experiments must therefore be primarily applied to a determination of the general motions of an airship under prescribed conditions, and all data that can be utilised for calculating such motions become fundamental to strength calculations. It appeared, therefore, that a suitable paper to present would be one dealing with all the aerodynamic data available from models and their applications to the problem of airship motion.

These problems, however, present considerable difficulties, but on making use of some of the results of experiments conducted on full-scale airships, great progress has been made during the three or four years following the Armistice, and some of the obstacles that prevented even approximate solutions of the problems of equilibrium and stability have to some extent been overcome.

1.1. One of the questions to be dealt with at the outset is that of "scale effect." The difficulty that arises when using the results of experiments on small models applies not only to airships, but to aircraft in general, though probably not to the same extent in the case of aeroplanes as in the case of lighter-than-air craft. In modern wind channels, models of aeroplanes to a scale of one-fifth can be experimented upon, whereas in the case of the airship it is not uncommon for experiments to be conducted on models whose scale is one two-hundredth of the actual ship.

1.2. Further, the inertia effects of the air, carried along with the envelope of an airship, may be considerable in comparison with the inertia of the airship itself, whereas in the case of heavier-than-air machines they can be neglected without serious error. Ignorance as to the magnitude of these two effects alone renders an accurate comparison of full-scale performance in accelerated motion with any prediction from model data exceedingly difficult, and might lead to erroneous conclusions. It is proposed later to show how, given appropriate experiments on models and full-scale, these effects can be estimated with greater accuracy than at present

1.3. A third difficulty arises out of the effects of rotation on the aerodynamic forces brought into play. Wind channel experiments in general deal with wind forces in translational motion only, though in certain cases the effects of rotation can be determined. No experimental method has been developed for measuring in a wind channel one important factor involved in an analysis of the motion of an airship in circling flight, and in stability investigations. This factor, usually denoted by Y_r, is the rate of change with angular velocity of turning, of the wind force acting perpendicularly to the axis of the ship. (Lateral force.) It is probable that this quantity could be determined by mounting a model on a whirling arm, but no facilities have hitherto been available in this country for such experiments. The method has been used in Italy—in fact, evidence of the importance of Y_r was furnished by the results of experiments conducted in Italy along these lines.

Theories have, however, recently been formed whereby an approximate estimate of this quantity can be made using model data only, but the method was only discovered on examining the results of experiments on full-scale—an indication of the importance of obtaining as much reliable full-scale information as possible. With such methods available the total forces acting on an airship can be determined by compounding the aerodynamic forces due to rotation with those due to translation. Thus it will be possible to investigate the general

aerodynamic characteristics of the airship—as far as the present information regarding scale and inertia effects permits.

1.4. For convenience then, we may first of all consider the aerodynamic forces directly measured in a wind channel, viz., the forces due to translation, and endeavour to analyse them into their component parts, that is, to examine how much the hull, fins, cars, etc., contribute to the total. It is not proposed to give more than a brief sketch of the experimental methods employed in wind channels for the determination of these quantities.

Attention will then be directed to an examination of the forces due to rotation, and thirdly to the combined forces and the determination of the positions of equilibrium in curvilinear flight with rudders inclined at various angles.

Fourthly, a consideration of the distribution of normal pressures over the hull and fins in steady motion will be given.

After having dealt with problems which are concerned with steady motions of an airship, the questions of stability and disturbed motion will be discussed. This must be preceded by a few remarks concerning inertia coefficients or acceleration effects.

It should be noted here, that the data and analysis presented in the following pages have been collected from reports prepared at the National Physical Laboratory for the Advisory Committee for Aeronautics and later the Aeronautical Research Committee. A list of the reports is given at the end of the paper, and footnotes indicate the particular reports to which reference is made. A few reports are added as appendices. It has been the writer's privilege to have collaborated in the preparation of the majority of these reports, some of them represent his own independent work. Further, he wishes to express his thanks to the Aeronautical Research Committee for permission to include in the paper information which has not yet appeared in the published reports of the Committee, but which however has been passed for publication. He is also indebted to the Executive Committee of the National Physical Laboratory for permission to present the paper.

II. *Notation*

2.0. *Axes.*—The origin of co-ordinates O is generally taken to be situated at the centre of gravity of the airship. The equations of motion, considered in stability theory, refer to a system of axes through the C.G., which is usually below the centre of buoyancy or the centre of volume of the hull. If the airship is in trim, *i.e.*, with its axis horizontal in static equilibrium, the C.G. is vertically below the C.B. For motion in a horizontal plane it is sufficiently accurate to assume them to be coincident, and in the approximate forms of the stability criteria considered in section XI. the terms involving the distance between C.B. and C.G. disappear in general. We therefore assume the origin to be situated at the C.B. when dealing with the data derived from experiments on models, since pitch and yaw then become interchangeable by symmetry. The appropriate correction can be applied where necessary.

The axis of x, under this assumption, lies along the axis of the hull, positive direction forward.

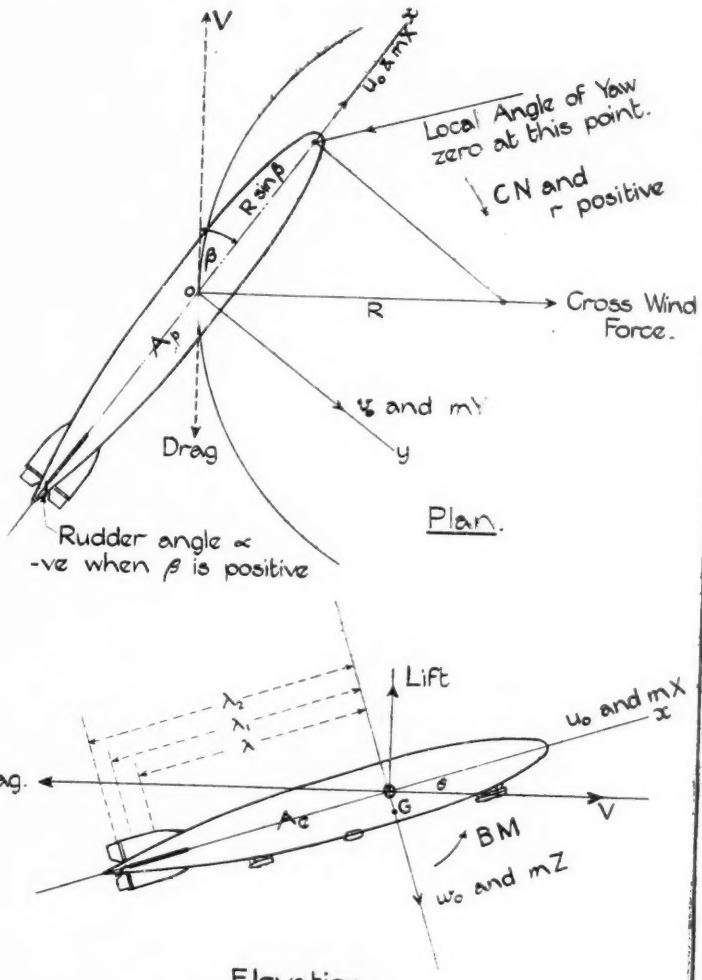
The axis of z is in the plane of symmetry perpendicular to OX, positive direction downwards.

The axis of y is perpendicular to the plane ZOX, positive direction to starboard, *i.e.*, to the right when looking along OX.

The system of axes is thus a right-handed system fixed in the airship. (See Fig. 1.)

2.1. V is the resultant velocity of the origin.

FIG. 1.


V
u_0 & mX
Local Angle of Yaw zero at this point.
CN and r positive
R sin β
β
o
R
Cross Wind Force.
A_p
Drag
v and mY
y
Plan.
Rudder angle α
-ve when β is positive
Lift
λ_2
λ_1
λ
u_0 and mX
x
Drag.
θ
G
A_e
V
BM
w_0 and mZ
Elevation.

u_o, v_o, w_o are velocity components along OX, OY and OZ respectively in steady motion.

u, v, w are small increments to u_o, v_o and w_o in disturbed motion.

p_o, q_o, r_o are components of angular velocities about OX, OY and OZ respectively in steady motion. p_o and q_o are of course zero in the case of an airship.

p, q, r are small increments to p_o, q_o and r_o in disturbed motion.

2.2. m is the mass of the airship.

Vol or l^3 denotes the displacement of the airship.

ρ is the density of the air at ground level, 0.00237 slugs/cu. ft.

$m=\rho l^3$ when the airship is in static equilibrium.

mX, mY and mZ are the components along OX, OY and OZ respectively of the resultant aerodynamic force acting on the airship. These forces are called longitudinal, lateral and normal force respectively.

A, B and C are the moments of inertia of the airship about the axes OX, OY and OZ respectively.

AL, BM and CN are the components (about OX, OY and OZ) of the resultant aerodynamic couple. The couples are termed rolling, pitching and yawing moment respectively.

$mX_o \ldots AL_o$, etc., are the values of $mX \ldots AL$, etc., in steady motion.

X_u, etc., $=\partial X/\partial u$, etc., and are termed resistance derivatives.

In disturbed motion, in which $u \ldots r$ are small and their squares and products negligible,

$$X=X_o+uX_u+vX_v+wX_w+pX_p+qX_q+rX_r$$
$$+\dot{u}X_{\dot{u}}+ \qquad\qquad +\dot{r}X_{\dot{r}}$$

where $X_{\dot{u}}$, etc., $=\partial X/\partial \dot{u}$, etc. The derivatives $X_{\dot{u}}$, etc., may be termed "acceleration derivatives" or "inertia coefficients."

2.3. β is the angle of yaw, *i.e.*, the inclination of the axis OX to the direction of motion of O in the plane XOY.

θ is the angle of pitch, *i.e.*, the inclination of the axis OX to the direction of motion of O in the plane XOZ.

mY_β and CN_β are the lateral force and yawing moment in pure translational motion at an angle of yaw β.

mZ_θ and BM_θ are the normal force and pitching moment in pure translational motion at an angle of pitch θ.

Suffixes h, f and r with brackets will be used to denote due to hull, fins and rudders respectively, thus $(mY_\beta)_h$ = lateral force due to hull in pure translational motion at an angle of yaw β.

2.4. R (i.) denotes resistance in sections III. and IV.

(ii.) denotes the radius of the path of O in steady circling flight.

λ is the distance of the centre of pressure of the forces on the fins from the C.B.

λ_1 is the distance of the centre of area of fins and control surfaces from the C.B.

λ_2 is the distance of the hinge of the control surfaces from the C.B.

These three quantities are the absolute values of the distances.

(λ is also used in the theory of stability in the expression $e^{\lambda t}$.)

α is the inclination of the control surfaces to the fixed fins.

A_e and A_p denote the longitudinal area of the hull in elevation and plan respectively.

A_f denotes the area of fixed fins and control surfaces, vertical or horizontal, but not the sum of both.

A_r denotes the area of control surfaces, vertical or horizontal.

III. *The Law of Dynamical Similarity in Steady Motion*

3.0. The application of the results of experiments on aircraft models to full-scale performance is based upon the law of dynamical similarity. From this law it may be deduced that up to the speeds in the neighbourhood of the velocity of sound, the motion of a fluid past two similar obstacles will be similar when the product of the speed V and a linear dimension L is the same in the two cases, provided that the two bodies are "completely submerged" in the same fluid. If the fluids be different in the two cases, the criterion for similarity of motion is given by the product VL divided by the kinematic viscosity (ν) of the fluid. This law can be conveniently deduced by means of the dimensional theory as follows.*

3.1. The resistance R of a body in steady motion through a fluid can only depend on its size, shape, speed and the characteristics of the fluid. Since we are considering the application of results on one body to another body of similar shape, we may suppose the size to be determined by a linear dimension L. The characteristics of the fluid are defined by ρ and μ, the density and coefficient of viscosity respectively. Further, the speeds with which we are concerned are less than that of sound, and therefore the compressibility of the fluid under consideration, air, does not enter into the problem. Thus R can be expressed as a function of V, L, ρ and μ.

$$R = F(V, L, \rho, \mu).$$

Let any term in R be of the form

$$V^w L^x \rho^y \mu^z$$

the dimensions of this expression must be that of a force, *i.e.*, ML/T^2, or

$$\left(\frac{L}{T}\right)^w L^x \left(\frac{M}{L^3}\right)^y \left(\frac{M}{LT}\right)^z = \frac{ML}{T^2}$$

Hence each term in R will be of the form

$$\rho V^2 L^2 \left(\frac{VL}{\nu}\right)^{-z}$$

where ν is μ/ρ, the kinematic viscosity.

z cannot be determined from the above relations, and hence R must be expressed in the form $\rho V^2 L^2 f(VL/\nu)$, the function f depending on the shape of the body and manner of presentation. In cases where the forces are said to follow the square law, the function f is a constant for any particular attitude of the model.

3.2. In cases of experiments for which VL/ν is the same for the model as for the full-scale body to which the model results are to be applied, the forces on the full-scale body are immediately calculable. The difficulty arises when such is not the case. The use of a different fluid medium has been considered, and attempts are at present being made to extend the range of VL/ν to which model experiments have been confined, by the use of water instead of air.† But though in this case ν is decreased to about one-thirteenth of the value for air, the experiments are generally conducted at a lower speed and the extension is small, and the full-scale value of VL/ν is again so far distant as to render the application of the results uncertain. The uncertainty is all the more marked in a great number of cases in view of the fact that $f(VL/\nu)$ undergoes an appreciable

* See Aeronautics in Theory and Experiment, Cowley & Levy, Ch. iv.

† Another method under consideration involves the use of a high pressure channel.

variation with speed within the range obtainable in a wind channel. However, at the higher values of VL, the resistance coefficient of a streamline model varies much less rapidly with speed and in some cases shows definite signs of becoming constant. It is to be understood in this connection that the resistance coefficient of an airship model is defined as $R/\rho v^2 l^2$ (l being the cube root of the volume of air displaced), and consequently is equal to $f(vl/\nu)$. This definition has been adopted as giving a comparison of different airship forms on a basis of equal lift, lift being roughly proportional to the volume. We will now proceed to consider the head resistance of an airship model in detail.

IV. *Head Resistance of Airship Models*

The method* in general use for measuring the resistance may be briefly described as follows. The model is suspended by two fine steel wires from the roof of the channel. A small sting is inserted in the after end of the model, and this sting pushes against a small conical point in a streamline spindle supported in the chuck of the aerodynamic balance. The spindle is shielded from the wind as far as possible by means of a suitable guard. The resistance of the wires is measured by duplicating them by the addition of similar wires to the floor; the difference between the resistance of the model plus four wires, and that when two wires only are attached gives the effect of the wires. The interference of one pair of wires on the other is so small that it can be neglected.

The resistance of the spindle is observed by detaching the sting from the spindle and supporting the model in the correct relative position.

Allowance has to be made for the effect of the variation of static pressure† in the wind channel, due to the acceleration of the current towards the airscrew and friction between the air and the walls. The total resistance obtained, after applying corrections for wires and spindle as above, is the true resistance plus a spurious resistance due to this pressure variation. It is sufficiently accurate for the purpose of applying this correction to take the pressure variation to be that in the undisturbed channel, *i.e.*, in the air current in the absence of the model.

4.1. A comprehensive series of experiments‡ was conducted at the N.P.L. by the late Mr. Pannell and the writer on the effect of form on resistance. A series of composite models was used, the maximum diameter being six inches, and as the series consisted of four alternative forms of heads, five tails and a number of cylindrical portions of varying lengths, a variety of forms could be built up, and the effect of altering the head, tail or length of cylindrical body could be examined.

It is obviously impossible to include in a paper of this nature all the results given in the report describing these experiments. Fig. 2 shows, however, the models used, and a few typical curves illustrating the main features of the results are reproduced in Figs. 3 and 4.

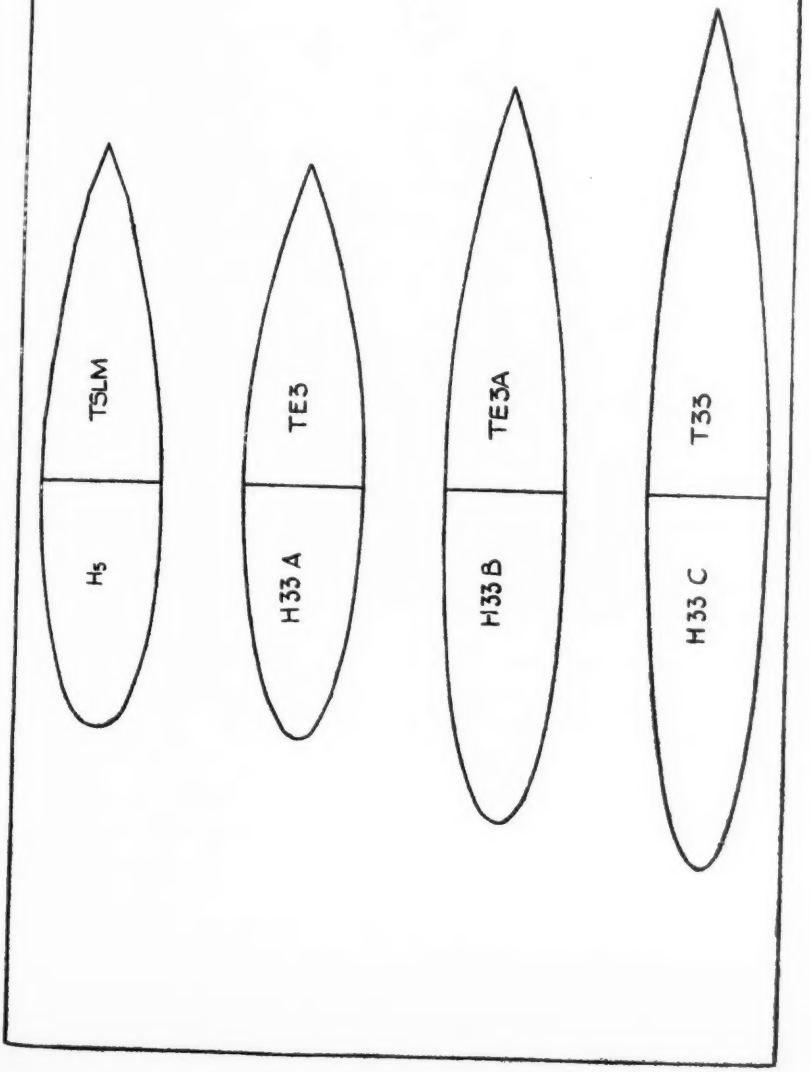
Notes on the Models

4.2. (*a*) *Heads.*—The selection of the heads was based on the results of previous experiments§ on models which appeared to be more or less efficient from the point of view of resistance. It should always be borne in mind that an extremely desirable characteristic of an airship of given length and maximum diameter is that the capacity should be the maximum consistent with low resistance. Evidence from the experiments referred to showed that a "good head is one which has a small average curvature and merges very gradually into the parallel portion." Further, the resistance coefficient of the airship R.33‖ was

* R. & M. 244 (see list of Reports).
† R. & M. 607 (see list of Reports).
‡ R. & M. 607. § R. & M. 246 and 311. ‖ R. & M. 361

FIG. 2.

Examples of Models of Airship Hulls § 4·3



low, and the form of the head of that airship was approximately elliptical. Accordingly, it was decided to make use of a model head similar to that of R.33. This model, H.33A of Fig. 2, terminates at the position where the diameter was one per cent. less than the maximum. This gave rise to a small discontinuity at the junction with the cylindrical body or the tail and model H.33B was made slightly longer, so that the tangent to the curve at the after end was parallel to the axis. In order to examine further the effect of increasing the length of the head, model H.33C was designed by expanding the abscissæ of the head H.33A, in the ratio of 3 : 2.1, leaving the ordinates unchanged. H.5, the other model head, was the best head used in experiments conducted previously to those on R.33 (R. and M. 246).

(*b*) *Tails.*—As in the case of the heads, previous experiments formed a basis for the selection of the tails. T.33 is the tail of R.33, SLM tail is that of R.32, T.23 is that of R.23 and E.3 one that appeared to be efficient from the results of R. and M. 311. E.3A is of length intermediate between T.33 and E.3 and designed by expanding the longitudinal dimensions of the latter. The curve of SLM. is not continuous, but consists of a series of straight lines.

(*c*) *Parallel Body.*—Several cylinders, whose length/diameter ratios varied between $\frac{1}{2}$ and 5, were used in conjunction with each combination of head and tail. C_1 C_2 represent lengths equal to one and two diameters respectively, and so on.

Effect of Varying the Head

4.3. Fig. 3 gives a general idea of the effect of using different heads on the same combination of cylindrical body and tail. The four curves in each three sets show the resistance coefficients plotted against VL, when each of the four heads is used with (i.) T.33, (ii.) C_1 and T.33, and (iii.) C_2 and T.33. It would appear that the finer the head, and thus the more gradual the rate of change of curvature, the lower the resistance coefficient at the higher speeds. It will be noted that at the lower values of VL the coefficient varies rapidly with VL, showing that $f(VL/\nu)$ is by no means constant, and that it would be extremely dangerous to draw conclusions regarding the resistance of full-scale airships from experiments at low values of VL on models. The curves become flatter at high speeds and the coefficient tends to become constant, but there is no evidence to prove that at some higher value of VL it will not again change.

Effect of Varying the Tail

4.4. Fig. 4, in which is plotted the resistance coefficient of head H.33B when used in conjunction with combinations of T.SLM, TE.3, TE.3A, T.33 with C_0, C_1 and C_2, does not indicate any consistent change as the tail is changed. Tails of the same over-all dimensions as that of T.33, but whose cross sections were elliptical, have been tested when used in conjunction with the R.33 head and cylindrical body. The resistance coefficient at high speeds was but very slightly higher than that of R.33 and appeared to justify the adoption of such a modification in future design, particularly in view of the fact that the strength could be increased and the area (and therefore weight) of the stabilising surfaces decreased.

Effect of Cylindrical Body

4.5. If the lower values of VL be ignored, it is evident from comparing the three groups of curves in both Figs. 2 and 3 that the resistance coefficient increases with the length of cylindrical body. A close examination of the results

FIG. 3.

RESISTANCE COEFFICIENTS OF MODEL AIRSHIP HULLS.

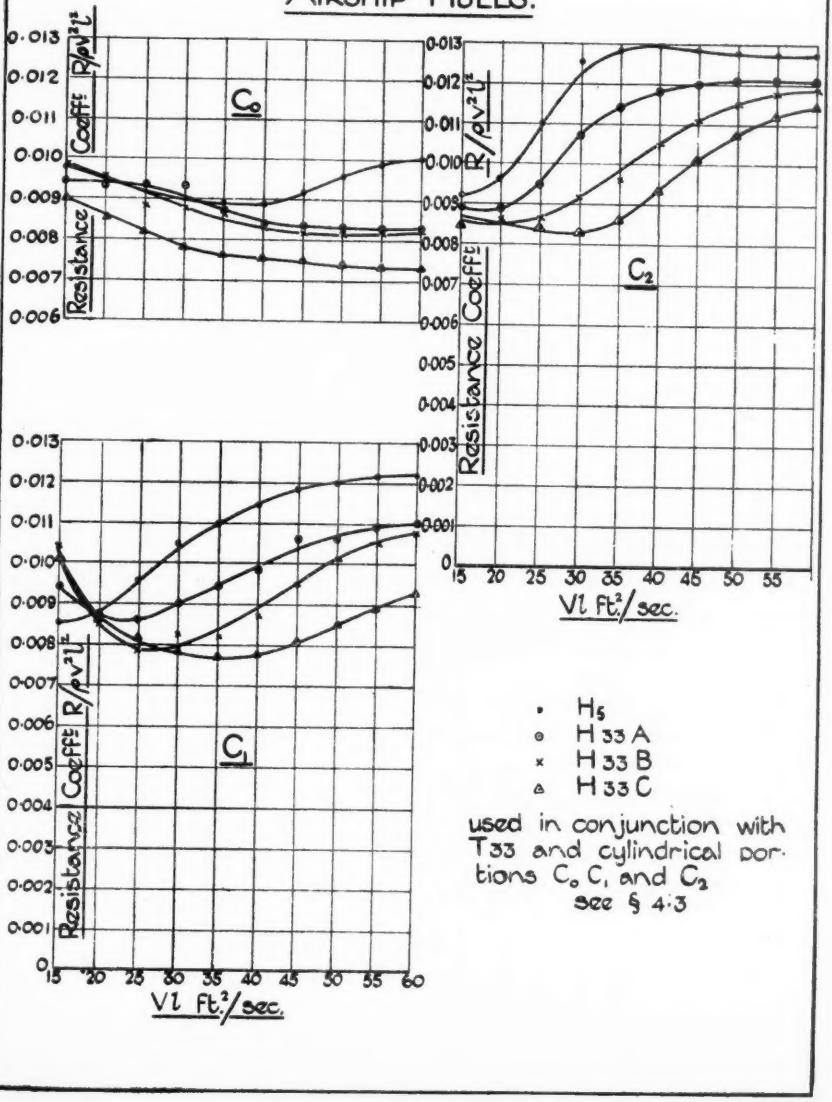
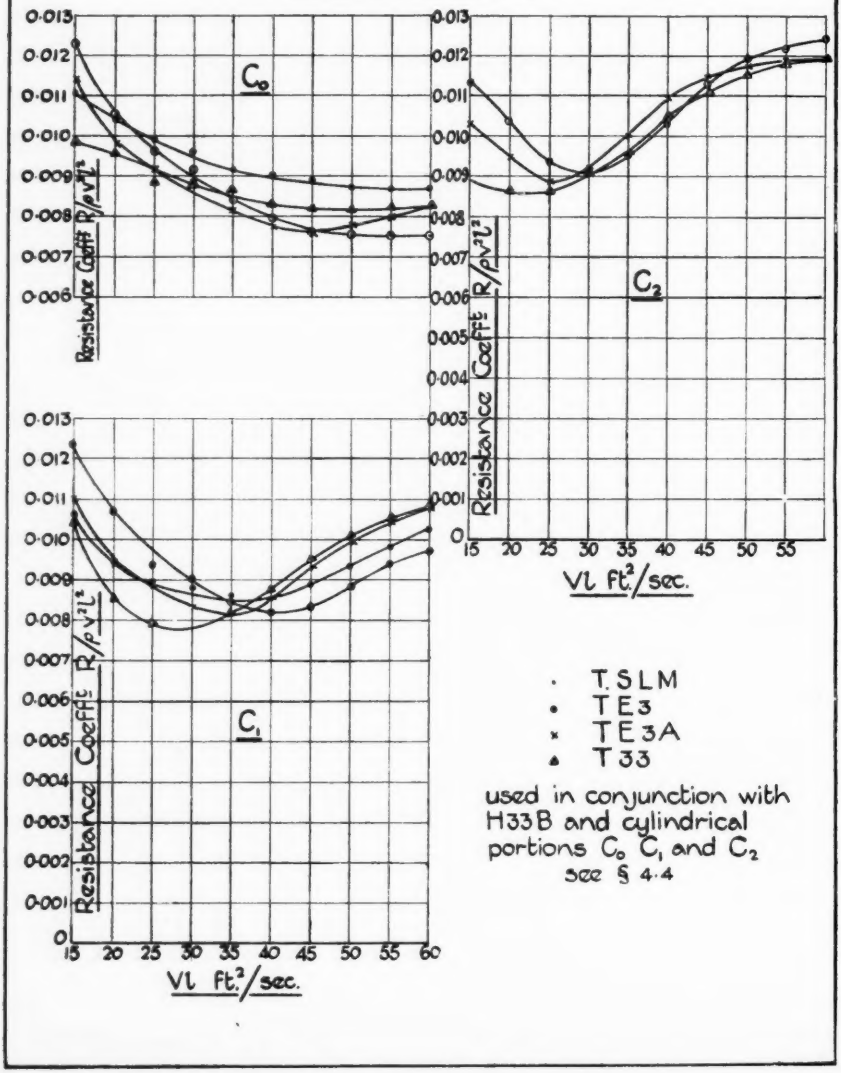


FIG. 4.

RESISTANCE COEFFICIENTS OF MODEL AIRSHIP HULLS.



shows that the increase in resistance coefficient may be accounted for by assuming the resistance of the cylindrical body to be entirely due to skin friction.

4.6. Broad conclusions may be stated as follows:—

1. A tail 2.5 diameters long can be made to give as low a resistance coefficient as any other tail yet examined.
2. A form 4.6 diameters long can be produced which will give a resistance coefficient of 0.007, comparing with 0.013 for "S.S.," 5.2 diameters long, and 0.011 for R.33, 8.2 diameters long. The forward curved portion of such a body may be elliptical and must be at least 2 diameters long. There should be no cylindrical portion.
3. For all the models examined, except in a few isolated cases, the introduction of cylindrical body causes an increase in resistance coefficient at the higher speeds.
4. In forms which include more than one diameter of cylindrical body, a tail composed of a series of truncated cones and 2.75 diameters long may be used without appreciable increase in the resistance coefficient at the higher speeds.

Form of the Resistance Curves at Low Speeds

4.7. Recent experiments* conducted on an approximately streamline model with fine wires placed around the head in a plane perpendicular to the axis, show that as the magnitude of the disturbance thus introduced increases the resistance coefficient increases at high speeds, but it may decrease at low speeds. In all cases the variation of the coefficient with speed is less regular and the coefficient has a definite minimum at a certain speed which becomes less as the magnitude of the disturbance increases, or in other words, as the final resistance coefficient increases. This phenomenon is also noticed on examining Fig. 3, and if disturbances are introduced into the air current forward of the model the same change in the nature of the curves is observed. The suggestion is offered that the motion is not steady at the low value of VL and that possibly time enters explicitly into the function of VL and ν upon which the resistance was seen to depend (§ 3.1).

Skin Friction

4.8. It was mentioned above that the increased resistance due to cylindrical body was due to skin friction. It is probably true to state that the resistance of the complete body is also due to skin friction.

Formerly it was usual to assume that the resistance of a streamline body consisted of two parts, termed "form resistance" and "skin friction resistance." The form resistance arises from the integration of the normal pressures on the body, but experiment has shown that if the correction be applied for the variation of static pressure the integral vanishes or at least becomes very small.† Thus, the resistance would be due entirely to skin friction, and it is easy to show that if a constant skin friction coefficient be assumed the resistance should be proportional to the area of the axial section in the case of a solid of revolution. This is, however, not the case, hence we conclude that either the skin friction coefficient varies with the form of the body, or that the form resistance is not zero in all cases.‡

The Resistance of Complete Models

4.9. Two methods present themselves for obtaining the resistance of a complete airship model. The first and most direct is to attach small model cars

* R. & M. 858.

† R. & M. 600 and §9.2.

‡ The form resistance is not zero in the case of a prolate spheroid, the ratio of whose axis is 4.1.

and fins to the hull and determine the resistance of the whole. The second method* is to measure the resistance of large models of the cars in the absence of the hull, and add the result to the resistance of the hull and stabilising surfaces. The resistance of car supporting wires and fin bracing wires, radiators, etc., can in this case be calculated and the total resistance obtained.

An objection that has been advanced against the first method is that the model cars being so small, VL effect may be great even at the highest channel speed, but it is probable that since the car resistance is only about one-fifth of the total, any "VL effect" that may influence the cars will not seriously alter the result. In favour of the method it may be pointed out that any interference between cars and hull is included in the measurement, whereas in the second method the interference effect has to be determined separately. Experiments have, however, shown that the interference of hull on the cars may be neglected and the interference of cars on the hull is small.

A table is given below showing the most important results available at the highest VL in the wind channel, together with a comparison with the resistance coefficient obtained from deceleration trials on full-scale. With regard to the latter it should be mentioned that inertia effects are present in accelerated motion, and no reliable evidence regarding these effects is available, though it is probable that in head-on motion the effect is probably equivalent to increasing the mass of the ship, and hence the resistance coefficient by about 10-20 per cent. The question is considered in greater detail in section X.

First Method

	Resistance Coefficient.	Model Results. Resistance as Percentage of Totals.			Full Scale	Model.
Airship.	Complete Model.	Hull only.	Cars and Supports.	Stabilising Surfaces.	Resistance Coefficient.	Full Scale.
R.26	0.0246	—	—	—	0.0248	0.99
R.29	0.0232	73	14	13	0.0227	1.02
R.32	0.0188	74.3	20.4	5.3	0.0199	0.95
R.33	0.0166	68.9	22.7	8.4	0.0173	0.96
R.38	0.0178	71.3	23.4	5.3	—	—

Second Method

	Predicted from Model Experiments. Resistance as Percentage of Total.									
						Wires.				
Airship.	Resistance Coefficient Complete Ship.	Hull.	Fins.	Cars.	Fin.	Control Bracing.	Car.	Various.	Full Scale Resistance Coefficient.	Model. Full Scale.
R.26	0.030	62*	6.2	19.8 (3)	3.0	0.9	1.6 (3)	6.5	0.025	1.20
R.33	0.0197	69*		14 (4)	3.2	0.8	3.0 (4)	10.0	0.0173	1.14

* Includes interference of cars on hull.

It will be seen that, using the first method, good agreement obtains between model and full-scale results. It is possible that by a remarkable coincidence scale effect, inertia effects and errors introduced by failure to reproduce various items such as fin bracing wires, car supports and radiators on the model almost cancel each other. On the other hand, an allowance for inertia effect of the magnitude it is believed to be, would bring the results obtained by the second method into closer agreement with full-scale. Further full-scale experiments, with a development of a method for determining the airscrew thrust accurately, would be of inestimable value in elucidating the points concerning which no reliable information is at present available.

* R. & M. 619 and R. & M. 827.

V. *Aerodynamic Forces and Couples on an Airship Model with the Axis Inclined to the Direction of the Relative Wind*

5.0. It was pointed out in § 1.3 that direct wind channel measurements in general deal with translational motion only. Such a motion may be an accurate reproduction of the actual motion of an airship, provided the angle of yaw is zero. For example, an airship may be in rectilinear motion and at the same time pitched if the elevators are set at the appropriate angle; this angle and the angle of pitch will be determined by the speed of the airship, the difference between the weight and the lift, the trim and the aerodynamic properties of the hull and stabilising surfaces.

If, however, the airship be yawed, it must of necessity turn, and turn at such a rate that the centrifugal force balances the aerodynamic cross wind force (*i.e.*, the force acting perpendicularly to the direction of motion, not to be confused with lateral force or the force perpendicular to the axis of the ship). Moreover, the yawing moment must vanish. It is evident that such a motion cannot be reproduced in the wind channel in the same manner as it could be on a whirling arm for example.

5.1. The cross wind force and yawing moment in curvilinear motion will not be the same as those in rectilinear motion, *i.e.*, those directly measured on the aerodynamic balance; both will now be functions of the angular velocity. Owing to lack of experimental evidence, the nature of the functional relation is difficult to determine accurately. The experiments* conducted in Italy with an airship model mounted on a whirling arm (referred to in § 1.3), show that the variation of the yawing moment with angular velocity is linear to a very close degree of approximation. The same experiments show that a similar relation exists between the cross wind force and angular velocity; the degree of the approximation in this case is by no means as good, but is probably sufficiently close since the rate of turn is always small. Let us assume then that the total cross wind force and yawing moment may be resolved into two components, one due to rotation and the other due to pure translation. Expressed analytically, let

$$CN_o = CN_\beta + r_o CN_r$$

and similarly for cross wind force.

In the above expression

CN_o is the total yawing moment in steady curvilinear motion.
CN_β is the component due to pure translation at an angle of yaw.
r_o is the angular velocity, and
CN_r is $\partial CN/\partial r$ and is proportional to V the resultant velocity.

It is also assumed that CN_β and the cross wind force due to translation are proportional to V^2. We shall see below (§ 6.1) how far this may be expected to hold good.

It is proposed first of all to discuss the nature of the forces due to pure translation with a brief note of the experimental methods employed in determining them, and later to direct attention to the rotational components.

VI. *Forces and Couples Due to Translation*

6.0. The usual quantities measured directly on the aerodynamic balance at the National Physical Laboratory are:—

(*a*) Lift, drag and pitching moments at various angles of pitch, the angle of yaw being zero. The model in this case is rigidly attached to a spindle placed perpendicularly to the plane of symmetry of the envelope, the centre line of the

* T.1320.

spindle passing through the C.B. of the model. The spindle is mounted on the chuck of the balance so that lift and drag can be observed simultaneously; a slight modification of the balance enables the pitching moment to be obtained. It has been found that a spindle projecting from the model does not in this type of experiment unduly affect the forces to be measured. The effect of the spindle is obtained by measuring the resistance on wires, as described in § 4.0, and comparing the result with the drag at zero pitch as measured on a spindle. It is usual to conduct the experiments on the bare envelope, envelope and cars, and finally on the complete model with the elevators set at various angles, the rudders being always set amidships. From these three sets of observations data are available for determining the effect of cars, stabilising surfaces and elevators.

(*b*) Cross wind force, drag and yawing moments at various angles of yaw, the angle of pitch being zero. In these experiments several rudder settings are used, the elevators being amidships. The model is now supported with the spindle placed in the plane of symmetry. Measurements similar to those described in (*a*) are usually made.

The four forces referred to above are directed along axes fixed in space, *i.e.*, along and perpendicular to the direction of the wind. It is, however, more convenient in stability calculations to refer these forces to axes fixed in the body. It is therefore customary to reduce the forces so that in reports describing experiments of this type, the longitudinal and normal forces are given at various angles of pitch, and longitudinal force and lateral force at various angles of yaw.

Since all airship hulls are approximately solids of revolution, the nature of the forces and couples on the envelope is not very different for pitch and yaw. Further, the vertical and horizontal fins are approximately similarly situated with respect to the axes of reference, so that fin effect is similar in both cases. The effect of cars is, however, obviously different.

Speed Effect

6.1. We have seen how in the case of the head resistance of an airship the drag does not in general vary as the square of the speed, particularly at the lower values of VL obtainable in the wind channel. The evidence regarding the variation of normal and lateral force with speed is much more scanty; reference might be made, however, to experiments conducted with a view to examining the speed effect on lateral stability.* In the case of the airship considered, it was found that the lateral force coefficient mY_β/V^2, where mY_β is the lateral force on the model at a speed V and angle of yaw β, increased by about five per cent. as V increased from 40 to 80 feet per second. The yawing moment coefficient CN_β/V^2, on the other hand, decreased by 2.5 per cent. for the same variation in V. The rate of variation, however, becomes much less at the higher speed in both cases. These results confirm previous conclusions obtained from experiments on a model of a non-rigid airship.† It would appear, therefore, that no serious error would be introduced by assuming these quantities to vary as V^2 and applying an appropriate correction where necessary. For convenience a wind speed of 40 ft./sec. is generally used.

Forces on the Hull

6.2. (i.) *Longitudinal Force.*—The variation of longitudinal force with the inclination of the axis of the model to the direction of the relative wind calls for little comment. The most important feature is its constancy at small angles, even up to about 10 degrees in some cases. It is not uncommon for the force to be slightly less at angles of incidence of one or two degrees, say, than it is at zero incidence.

* R. & M. 593. † R. & M. 457.

(ii.) *Lateral and Normal Forces.*—Owing to symmetry the lateral force is always zero at zero yaw, but such is not accurately the case for the normal force at zero pitch on a model airship of polygonal cross section, since the polygon usually has an odd number of sides, one side being bisected by the plane of symmetry, *i.e.*, *XOZ*. The variation of either force with angle of incidence is gradual up to an angle of about 6 degrees and then becomes rather more rapid. After an examination of the results of experiments on several models it has been found that the lateral force coefficient $(mY_\beta)_h/A_e\rho V^2$ does not vary very greatly from ship to ship. A_e is the longitudinal area of the hull in elevation. The same remark applies to the normal force coefficient $(mZ_\theta)_h/A_p\rho V^2$, where A_p is the plan area.

Tabulated results on a selection of models of the most recent airships are given in Appendix I., Table 1. As as illustration of the variation of the force coefficient with angle of incidence, the mean curve obtained from the results on several models is reproduced in Fig. 5.

It must not be assumed that this coefficient is an absolute constant even at one value of VL; it is difficult, however, to draw hard and fast rules for its variation from one model to another. The one case in which there appears to be regular variation with the form of the hull is when the head and tail remain the same, but are separated by different lengths of cylindrical body. Evidence on the point is scanty, but experiments conducted on a model of a contemplated airship of rather unique design* indicate that as the length of cylindrical body inserted increases by an amount equal to the diameter of the model the force coefficient decreases by about 18 per cent. of that on the model with no cylindrical body.

(iii.) *Yawing and Pitching Moments.*—The remarks made at the beginning of (ii.) regarding lateral and normal forces apply to yawing and pitching moments. The curve of moment against angle of incidence differs from that of force in one essential feature. The slope of the latter increases numerically with angle of incidence (up to an angle of 20 degrees at least); the slope of the former decreases numerically. Thus, the centre of pressure of the force on the hull approaches the centre of buoyancy as the angle of incidence increases. At zero incidence it is well forward of the C.B., but within the range of angles of incidence considered $(-20° < o < 20°)$ it never appears aft of the C.B.

The moment coefficient analogous to the force coefficient discussed above is $(CN_\beta)_h/\rho V^2 vol$ or $(BM_\theta)_h/\rho V^2 vol$ (yawing and pitching respectively) where *vol* is the volume of the model. This coefficient does not in general vary rapidly from model to model, and Fig. 6 shows the mean value comparable to the mean force coefficient given in Fig. 5. The only regular variation of coefficient with form hitherto met with is again the variation with length of cylindrical body. In this case corresponding with the 18 per cent. decrease in the force coefficient there is an increase of about 6.5 per cent. in the moment coefficient. Tabulated results on a selection of models are given in Appendix I., Table 2.

Effect of Cars

6.3. The lateral force on a model with cars attached is naturally greater than on the model with no cars. The increase varies between wide limits according to the type of hull and the shape of the cars. For example, let us consider models of R.29, R.32 and R.38. Taking these in order, the lateral force due to cars (*i.e.*, the difference between the lateral force on the hull with cars attached and that on the bare hull) is 70, 32 and 20 per cent. of that on the hull only at small angles of yaw. At large angles of yaw the corresponding percentages are

* Tail section elliptical. See § 4.4. Models C, D and E of Appendix I. See also Fig. 6, App. I.

40, 22 and 10 respectively. Of these three models R.29 has the longest cylindrical body, R.32 comes a close second, and finally R.38, so that, *cæteris paribus*, the lateral force coefficient on R.29 could be expected to be less than on the R.32, and again on R. 32 less than on R.38. This is actually so (see Table 1, Appendix I.).

It would thus appear that as design progressed and the head resistance of cars was reduced, the lateral force on the cars was also reduced.

The effect of cars on the yawing moment coefficient is hardly noticeable owing to the fairly uniform distribution with respect to the axis of z.

Normal force and pitching moments are also practically unaffected by cars; this is probably due to the shielding effect of the body.

Fin Efficiency

6.4. The criterion for fin efficiency is taken to be the lateral force due to the fin with rudders amidships (or normal force, with elevators amidships) divided by $\rho V^2 A_f$. A similar criterion is obtained by considering the moment due to fins, divided by $\rho V^2 \lambda_1 A_f$, where A_f is the area of fixed fins and stabilising surfaces on which the force acts, λ_1 the distance between the centre of area of A_f from the C.B. and ρV^2 has its usual meaning. The force due to fins is defined in this instance as the difference between the force on the hull and cars in the absence of fins and the force on the complete model. Similarly for moments.

The results of experiments on a large number of models have been examined and though certain forms of fins appear to have certain characteristics and certain forms of hull appear to affect the fin efficiency in a particular manner, yet it is impossible from the evidence available to draw any definite conclusions. A few very general deductions may be drawn, viz., (i.) the greater the ratio of width to length of a fin the greater the efficiency of the fin; (ii.) the higher the resistance coefficient of the model hull the lower the fin efficiency; (iii.) fins whose section is of a V form so as to enclose all bracing wires and so shield them from the wind are less effective than flat fins of the same dimensions; and finally, (iv.) the form of the tail may have an appreciable effect.

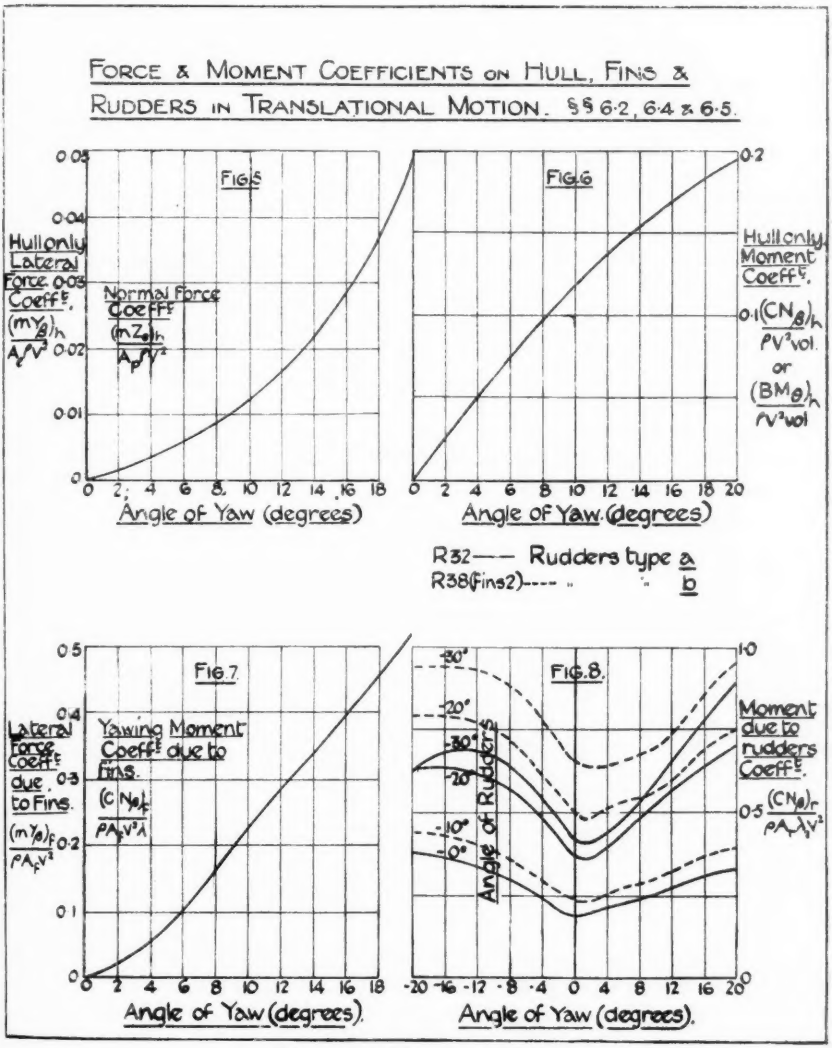
Table 3 of Appendix I. gives the value of the absolute force coefficient given above for several fins, but in order to give a graphical illustration of the nature of the variation of the coefficient with angle of yaw or pitch, Fig. 7 shows the mean value obtained from the results of all models hitherto tested at the National Physical Laboratory.

An exactly similar curve is obtained for the moment coefficient; the ordinates, *i.e.*, the moment coefficients themeslves, are about 10 per cent. less than those of Fig. 7 at all the angles of yaw considered. This signifies that the centre of pressure of the forces on the fins with rudders amidships is situated forward of the centre of area of the fins by a distance equal to 0.1 λ_1, *i.e.*, $\lambda = 0.9\ \lambda_1$, and its position remains practically unchanged as the angle of yaw varies. It should be noted that the force due to fins is of the same sign as the force on the hull, whereas the signs of the moment due to fins and moment on the hull are different. Thus, the force curve for the combination of hull and fins becomes steeper, whereas the moment curve becomes less steep. Hence, the centre of pressure of the forces on the combination is further aft than that of the forces on the hull. It is found in general that the fins in common use are not sufficient to cause the centre of pressure to move aft of the centre of buoyancy for usual angles of incidence, and thus an airship would never be stable as a weathercock about an axis through the C.B.

Control Efficiency

6.5. A measure of the efficiency of the control surfaces is given by either of the following criteria :—

Force due to inclining the control surface* through a certain angle divided by the area of the surface and by ρV^2 or the corresponding moment divided by


FORCE & MOMENT COEFFICIENTS ON HULL, FINS & RUDDERS IN TRANSLATIONAL MOTION. §§ 6·2, 6·4 & 6·5.
FIG.5
Hull only Lateral Force Coeff^t. $\frac{(mY_\theta)_h}{A_p V^2}$
Normal Force Coeff^t $\frac{(mZ_\theta)_h}{A_p V^2}$
Angle of Yaw (degrees)
FIG.6
Hull only Moment Coeff^t. $\frac{(CN_\theta)_h}{\rho V^2 vol.}$ or $\frac{(BM_\theta)_h}{\rho V^2 vol}$
Angle of Yaw (degrees)
R32 —— Rudders type a
R38(fins2) ---- " " b
FIG.7
Lateral Force Coeff^t due to Fins. $\frac{(mY_\theta)_f}{\rho A_f V^2}$
Yawing Moment Coeff^t due to fins. $\frac{(CN_\theta)_f}{\rho A_f V^2 \lambda}$
Angle of Yaw (degrees)
FIG.8.
Angle of Rudders
Moment due to rudders Coeff^t. $\frac{(CN_\theta)_r}{\rho A_r \lambda V^2}$
Angle of Yaw (degrees).

$(mZ_\theta)_h$ *and* $(CN_\theta)_f$ *are negative.*

* The force due to inclining a control surface is defined as the difference between force on the complete model with control surface amidships, and that when the control surface is inclined.

the area of the surface, the distance of the hinge from the centre of buoyancy of the model (λ_2) and by ρV^2.

Numerical results for a series of models will be found in Appendix I., Table 4. The angle of incidence of the airship hull in the cases given is zero. These results include cases of two types of control surfaces.

(*a*) That used on R.29, R.32 and R.33. In this type of control surface the balancing portion, *i.e.*, the portion forward of the hinge, extends along the whole length of the control surface, the hinge being situated about one quarter of the width aft of the leading edge. There is thus a clearance gap between fixed fin and control surface with the latter amidships, and the gap increases as the control surface is inclined.

(*b*) That used on R.38. This type has the balancing portion concentrated at the outer edge of the control surface, the fixed fin being shaped accordingly. There is no gap between fixed fin and rudder or elevator except for about one third of the length of the latter. Further, it was found experimentally* that some advantage would be gained by cutting off the corner of the fixed fin immediately forward of the balancing area.

These two types were fitted on a model of R.33 and type (*b*) was found to be 35 per cent. more efficient than type (*a*). Removing the balancing area completely and modifying the fixed fin so as to fill in the space left vacant, thus eliminating the gap altogether, gave a still further increase in efficiency, the unbalanced rudder being 12 per cent. more effective than type (*b*). A rudder of this type is, however, impracticable owing to the enormous force necessary to operate it (see below § 6.6).

It has also been found that removing the fixed fin decreases the efficiency of the control surface (*cf.* R.29 and R.29b, Appendix I., *loc. cit.*). (In R.29b all the fabric was removed from one fixed vertical fin of R.29.) Further, a flat control surface used in conjunction with V-shaped fixed fin is rather less efficient than when the fixed fin is flat (*cf.* R.38, fins 1 and 2, Appendix I., Table 4). Finally placing the control surface at the extreme after end of a ship with a tail of elliptical cross section and ultimately faired down to a line naturally reduces the efficiency (*cf.* model D in yaw and pitch).

It should be observed that if the signs of the angle of incidence of the airship and angle of inclination of the control surface be the same, the signs of the force due to control surface and the force on the complete model with control surface amidships is the same and the signs of the moments different.

Fig. 8 gives specimen curves showing the variation of the force coefficient due to inclining the rudder with angle of yaw and angle of rudder. The curves refer to a 1/133.3 scale model of R.32† and a 1/200 scale model of R.38† at a wind speed of 40 ft./sec. The greater efficiency of rudders of type (*b*), fitted to R.38, is evident from these curves. It may be stated that provided the angle of inclination lies between $-20°$ and $20°$ for control surfaces of type (*a*) (or even between $-25°$ and $25°$ for type (*b*)), the force due to the control surfaces is approximately proportional to the inclination. Outside this region the force increases less rapidly with the angle until a certain angle is reached, after which it decreases.

Balancing of Rudders. Hinge Moments

6.6. There is but little direct information on the magnitude of the moments about the axis of a control surface on an airship. Reference may, however, be made to a series of experiments* made on a large airship model. The control surface, in this case a rudder, was detached from the model hull and supported

* R. & M. 653. † R.32 full curves, R.38 dotted curves of Fig. 8.

on the balance by means of a fine spindle in the usual manner. The airship model itself with fixed fins and the remaining control surfaces was mounted in the correct position relative to the rudder.

Rudders of type (*b*) only were examined and the ratios of the area of the balancing portion to that aft of the hinge were 0, 0.141, 0.212 and 0.283. Angles of yaw of 0°, 2° and 5° and angles of rudder inclination between $-30°$ and $+30°$ were considered. When the ratio of areas fore and aft of the hinge was 0.283 the hinge moment was practically negligible for inclinations between $-10°$ and $+10°$, showing that the rudder was balanced. It is, however, desirable to have a rudder rather under than over balanced, and consequently the balancing area in this particular case is probably excessive. In the other three cases the hinge moments on one rudder of dimensions approximating to that usual in airship design (viz., 198, 227 and 241 sq. feet) and at a speed of 60 feet per second or 41 m.p.h. are 596, 308 and 184 lbs.-ft.* corresponding to the above ratios of 0, 0.141 and 0.212 respectively. The hinge moment in these cases thus varies almost linearly with the ratio of the balancing area to that of the unbalanced rudder.

VII. *Forces and Moments Due to Rotation*

It has been indicated above (§ 6.0*b*) that from the symmetry of the airship form, pitch and yaw are approximately interchangeable; the real difference arises when considering the questions of equilibrium, stability and controllability. To avoid repetition, therefore, we will confine our attention to the longitudinal force, lateral force and yawing moment due to rotation in yaw. For convenience, attention will be directed in the first place to yawing moment.

Moment Due to Rotation

7.1. It has been assumed (§ 5.1) that the total yawing moment CN_0 may be expressed as the sum of CN_β the yawing moment due to pure translation, and $r_0 CN_r$ where r_0 is the angular velocity. Evidence has been cited which indicates that CN_r is independent of r. It remains to consider the value of and the method of determining $\partial CN/\partial r$ or CN_r. The corresponding quantity in pitch is BM_q.

The theory of the method employed has been described in various books and reports† and it appears superfluous to include it in the present paper.

7.2. The experimental method used may be briefly described as follows. The model is supported in the channel so that it is free to oscillate about an axis through the centre of buoyancy. For yawing motions this is the axis of z. Three wires from the points of intersection of this axis with the top of the model are attached to the roof of the channel, forming, as it were, an inverted tetrahedron. A similar system leads from the point of intersection of the axis with the bottom of the model to the floor. To avoid oscillations about a horizontal axis, two vertical wires equi-distant from the axis of z support the model. The model thus oscillates as a bifilar pendulum, the period of which, in still air, may be altered by adjusting the tension in the wires forming the tetrahedron. N_r is determined by observing the period of the oscillations and the logarithmic decrement of the amplitude (by means of a spot of light passing over a logarithmic scale) at different wind speeds. The slope of the curve showing the quotient of logarithmic decrement and period plotted against wind speed gives a quantity proportional to CN_r/V. A knowledge of the moment of inertia of the model

* Mean values of moments at 10° inclination of rudder. Approximately independent of angle of yaw.

† For example: Bryan—Stability in Aviation. Bairstow—Applied Aerodynamics. R. & M. 77. R. & M. 665.

and the restoring couple due to the system of wires when the model is turned through a certain angle is then sufficient to determine CN_r/V.

Experiment shows that within the range of speeds generally obtained in wind channels CN_r/V does not vary with speed, at any rate to any appreciable extent. Hence the total moment may be said to vary approximately as V^2 since it equals

$$CN_\beta + \frac{V}{R} \cdot CN_r$$

where R is the radius of the path. We have already seen that CN_β is probably approximately proportional to V^2.

7.3. With regard to the contributions of the hull and of the fins to the value of CN_r. V, the following example may be cited as showing that by far the greater part is attributable to fins in an airship of standard design.

Model.	Hull and cars only.	Complete model.
R.29 ...	—	-1.51×10^{-3}
R.29a* ...	-0.35×10^{-3}	-1.49×10^{-3}
R.29b ...	—	-1.23×10^{-3}
R.32 ...	-0.26×10^{-3}	-1.80×10^{-3}

On reducing the portion due to fins to the form of non-dimensional coefficient

$$-\frac{(CN_r)_f}{\rho\lambda^2 A_f \cdot V}$$

values are obtained which do not vary widely from model to model. λ is determined as in § 6.4 by dividing $(CN_\beta)_f$ by $(mY_\beta)_f$.

The following are the actual figures for yaw o°:—

Model.	Coefficient.
R.29	0.0043
R.29a	0.0041
R.29b	0.0039
R.32	0.0038
R.33	0.0038
C†	0.0046
D†	0.0042
E†	0.0037
D (pitch) V fins	0.0034

It would appear from the values given for models C, D and E that the coefficient decreases as the length of cylindrical body increases. C has no cylindrical body, D has about 0.596 d and E 1.505 d lengths of cylindrical body, where d is the diameter of the model.

Further experiments show that inclining the rudders does not appear to affect CN_r appreciably. This is confirmed by the Italian experiments referred to in § 1.3 and § 5.1.

Effect of Yaw

7.4. The following table indicates the nature of the variation of $-CN_r/V$ (model scale) with angle of yaw:—

Angle of yaw.	R.29.	R.29a.	R.29b.	C.	D.	E.
0	1.51×10^{-3}	1.49×10^{-3}	1.22×10^{-3}	0.88×10^{-3}	1.14×10^{-3}	1.40×10^{-3}
5°	1.61			1.27	1.43	2.07
10°	1.93	1.82	1.44	1.48	1.80	2.68

* R.29a is the same as R.29 with part of the fabric removed from the upper fixed fin (see Fig. 1, Appendix I). R.29b is the same as R.29 with all of the fabric removed from the upper fixed fin (see Fig. 1, Appendix I).

† See Fig. 6, Appendix I.

N_r is obviously independent of the sign of β, hence since there can be no discontinuity at $\beta = 0$

$$\frac{\partial N_r}{\partial \beta} = 0$$

Lateral Force Due to Rotation

7.5. Following lines similar to those of the preceding paragraph it will be seen that the lateral force in circling flight can be determined from a knowledge of Y_r. It is unfortunate that up to the present no experimental means of determining this quantity has been developed in England. When the problem of airship stability was first examined Y_r was considered to be negligible. It will be evident later (§ 11.4) that on such a supposition no airship would be stable or even approach neutrality in rectilinear flight, and as airships had successfully flown without undue manipulation of the controls it became obvious that the assumption was not justifiable. The first indication of the magnitude of Y_r was derived as a result of the analysis of the Italian report (§§ 1.3 and 5.1), which showed that for the airship under consideration the value was approximately $0.6\,V$. After many unsuccessful attempts at obtaining a reliable estimate, full-scale experiments on R.29 proved to be of considerable use in developing a method which, on being applied to other airships, gave results which agreed with the results of experiments on full scale. In view of the fact that this method is inter-related with the problem of determining the equilibrium of an airship in curvilinear motion, the discussion of the method will be included in the consideration of equilibrium which is given in §§ 8.1—8.6.

Longitudinal Force Due to Rotation

7.6. The longitudinal force due to rotation may also be said to depend on the value of $\partial X/\partial r$ or X_r, but as in the case of Y_r no experimental method has been developed for determining it. For approximate calculations of airship motion, it is probably unimportant; methods similar to those applied later to Y_r, however, show that X_r is roughly equal to $V \sin \beta$.*

VIII. *Equilibrium of an Airship*

Rectilinear Flight. Dynamic Lift

8.0. It is evident that an attitude of equilibrium for an airship moving with rudders amidships and elevators amidships is that of head-on flight. From symmetry there can be no lateral force or yawing moment. Moreover, the normal force and pitching moment are both of very small magnitude, and if the ship be in static equilibrium and in trim, the angle of pitch will be zero. Occasions frequently arise, however, when the ship may not be in static equilibrium. For example, after heavy rain or snow she may become heavy and the lift will not be sufficient to enable her to retain her height. On the other hand, on a long trip, the weight of the petrol consumed may cause the weight of the ship to become less than the lift. Leakage of gas and superheating also affect the equilibrium. Under these circumstances it may become necessary to make use of the dynamic lift of the airship by flying with the axis inclined to the horizontal, the plane of symmetry still remaining vertical. This use of the aerodynamic forces for maintaining equilibrium has the serious drawback, that if the inclination of the axis exceed a certain small angle, the drag is increased and consequently speed is lost. At an angle of 10 degrees, an angle not uncommon in practice, the speed may be decreased by as much as ten per cent. or more if the elevators be inclined at large angles.

* R. & M. 751, p. 8.

Given the necessary model data, viz., lift, drag and pitching moment at various angles of pitch and elevator settings, it is comparatively straightforward to calculate the positions of equilibrium of an airship for which the distribution of weight and other statical data are known, since steady symmetrical motion (*i.e.*, motion in a vertical plane) does not involve rotation.

In this connection an investigation of some interest is described in a report entitled, "Measurements of Normal Force and Pitching Moment on Rigid Airship R.33."* The force and the moment were calculated on the full-scale airship in different flying altitudes from a knowledge of the ballast released, petrol and oil consumption and the movement of personnel; gas lift and the effects of superheat were also considered. On comparing the results with corresponding model measurements it was found that the force measurements in most instances were in excess of the predicted values; the discrepancies are, however, not serious and for a number of later experiments at larger angles of pitch the agreement was extremely satisfactory. The discrepancies in the case of the pitching moments are less pronounced and in several instances do not exceed 5 or 6 per cent.

Curvilinear Flight in a Horizontal Plane

8.1. The two conditions to be satisfied in order that equilibrium may be established in curvilinear motion are, expressed in symbols,

$$CN_\beta + r_0 CN_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$mY_\beta + r_0 mY_r = \frac{mV^2}{R} \cos\beta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These two equations can be written in the form

$$CN_\beta + \frac{V}{R} CN_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

$$mY_\beta + \frac{mV}{R} (Y_r - u_0) = 0 \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

Equation (1) expresses the condition that the total yawing moment must vanish and equation (2) that the total lateral force must balance the y component of the centrifugal force. In these equations m, CN_β, CN_r and mY_β are known, the first from particulars of the airship, the last three from experiments on models. We have seen that Y_r is not known, and hence from the two equations we cannot determine R and β and so arrive at the attitude of equilibrium.

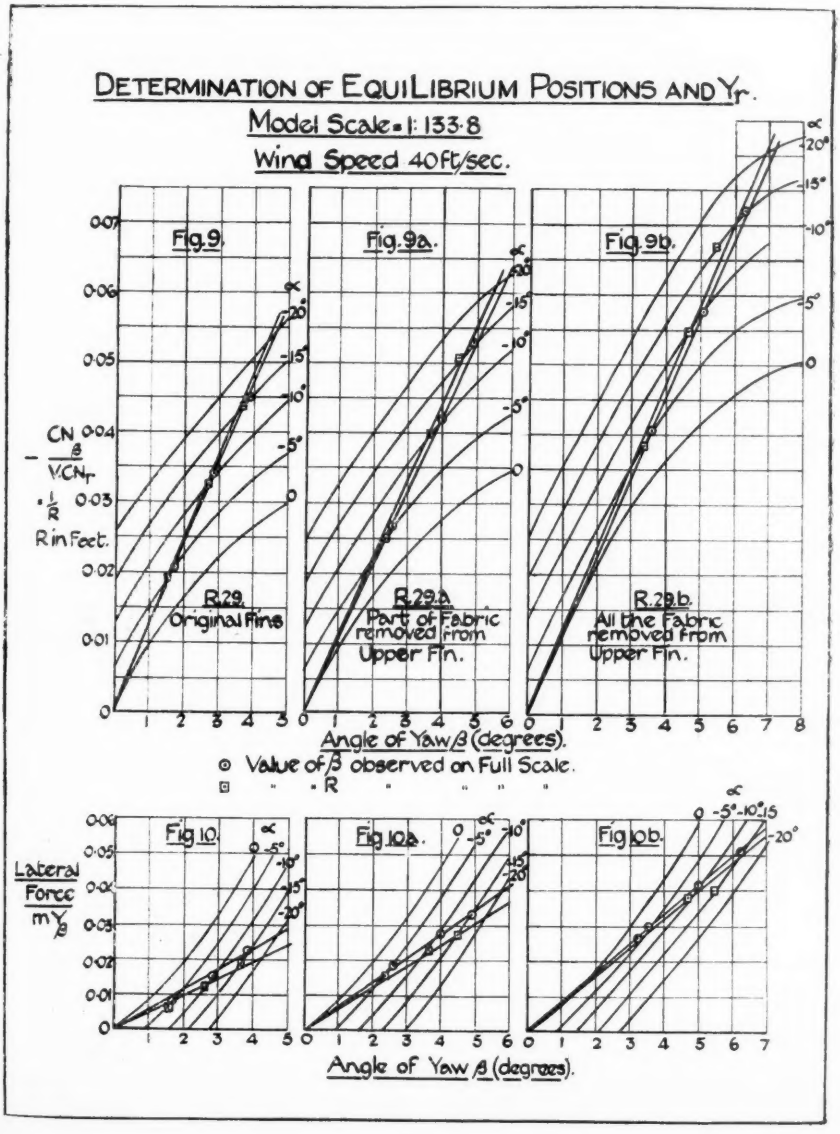
Suppose, however, that either R or β be known, then sufficient data are available to determine Y_r and either β or R. Equation (1*a*) alone suffices to find β if R be given or conversely. It thus forms a basis of comparison between the results of experiments on models and full-scale if full-scale values of R and β be known.

8.2. Let the quantity CN_β / VCN_r be plotted against β for various rudder angles α. Both CN_β and VCN_r having been assumed proportional to V^2, speed does not enter into the problem. Now from the results of experiments on full-scale the value of β for any particular α is known, let these values be marked off on the appropriate curve, and the values of $1/R$ for equilibrium are immediately available from the curves. See Figs. 9, 9a and 9b, which show the curves applicable to R.29, R.29a and R.29b.

The following table gives the values of R to model scale (1/133.3) calculated thus (R_c), and the values of R resulting from the experiments on full-scale (R_0). The values of α and β are also shown. The length of the model is 4.03 feet.

* R. & M. 815.

Rudder angle	R.29.			R.29a.			R.29b.		
α degs.	β degs.	R_c feet.	R_o feet.	β degs.	R_c feet.	R_o feet.	β degs.	R_c feet.	R_o feet.
5	1·75	48·3	51·5	2·6	36·9	39·1	3·6	24·7	25·5
10	2·9	29·3	30·8	4·0	23·5	24·9	5·1	17·2	18·3
15	3·9	22·2	22·8	4·9	18·9	19·8	6·3	13·7	15·0


DETERMINATION OF EQUILIBRIUM POSITIONS AND Yr.
Model Scale = 1: 133·8
Wind Speed 40 ft/sec.
Fig. 9.
Fig. 9a.
Fig. 9b.
R.29. Original Fins
R.29a. Part of Fabric removed from Upper Fin.
R.29b. All the Fabric removed from Upper Fin.
$-\frac{\beta}{V.CN_T} \cdot \frac{l}{R}$
R in Feet.
Angle of Yaw β (degrees).
⊙ Value of β observed on Full Scale.
⊡ " " R " " " "
Fig. 10.
Fig. 10a.
Fig. 10b.
Lateral Force / mVβ
Angle of Yaw β (degrees).

The maximum discrepancy between R_o and R_c occurs in the case of R.29b at $\alpha = 15°$ ($R_o = 1.09\, R_c$). It is possible that owing to the difficulty of reproducing on the model the bare girder work of the fin after removing the fabric, the model fin was not a sufficiently accurate representation of the full-scale fin. In the other cases the mean discrepancy is 5 per cent, $R_o > R_c$. Reference to § 6.1 will show that CN_β / V^2 was 2.5 per cent. less at 80 feet per second than at 40 feet per second for the model tested. If we allow a correction of 5 per cent. for speed effect on CN_β (the values of which used in compiling the above table are those of 40 ft./sec.) the discrepancy between R_c and R_o would be eliminated.

The agreement thus obtained between model and full-scale, even ignoring speed effect, is remarkably good and justifies a further analysis of the results with a view to obtaining a value of Y_r.

8.3. An examination of the curves yielding the above results showed that the curve $1/R$ against β for equilibrium is approximately a straight line through the origin, and moreover is roughly the same line in each of the three cases. Thus we conclude that $R\beta$ is approximately constant and independent of the fin area. This far-reaching relation provides a basis for a general method of determining Y_r and consequently led to an endeavour to find the factor deciding the value of the constant.

Let equations (1*a*) and (2*a*) be written in the form

$$(CN_\beta)_h + \frac{V}{R}(CN_r)_h + CN_f = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$(mY_\beta)_h + \frac{mV}{R}\{(Y_r)_h - u_o\} + mY_f = 0 \quad . \quad . \quad . \quad . \quad (4)$$

Now let

$$(CN)_f = -\lambda\,(mY_f)$$

This involves the assumption that

$$\frac{(N_r)_f}{(Y_r)_f} = \frac{(N_\beta)_f}{(Y_\beta)_f}$$

or that the position of the centre of pressure of the forces on the fins is the same in rotational motion as in translational motion. Such an assumption is justifiable since the main effect of rotation is to change the angle of incidence of the fins, and experiments show that $(N_\beta)_f / (Y_\beta)_f$ does not vary appreciably with β even at fairly high angles of yaw (up to about 20 degrees) (see § 6.4).

Also let $(Y_\beta)_h = \beta\,(Y'_\beta)_h$ and $(N_\beta)_h = \beta\,(N'_\beta)_h$ and eliminate Y_f and N_f from equations (3) and (4), then

$$R\beta = -\frac{m\lambda}{vol}\left[\frac{(Y_r - u_o)\,V + \dfrac{CN_r \cdot V}{m\lambda}}{\dfrac{A_e\lambda}{vol}\cdot\dfrac{mY'_\beta}{A_e} + \dfrac{CN'_\beta}{vol}}\right]_h \quad . \quad . \quad . \quad . \quad (5)$$

All the quantities in the square brackets refer to the hull and cars.

Now it has been stated that $(mY'_\beta / A_e)_h$ and $(CN'_\beta / vol)_h$ are approximately constant at the smaller values of β (up to 6 degrees or so) (§ 6.2, (ii.) and (iii.)). Also we have seen that $(CN_r)_h$ is small compared with CN_r; it may be said that $(CN_r / m\lambda)_h$ rarely exceeds 0.2 $\{Y_r - u_o\}_h$. It may be safely assumed for the present that $(Y_r)_h$ is small compared to u_o (in fact it is probably of the order of 0.1 u_o). It is positive and $(CN_r)_h$ is negative, so that the numerator of the expression in brackets in (5) is approximately $-u_o V$ or $-V^2 \cos\beta$ or $-V^2$ since we shall not deal with values of β greater than 6 or 7 degrees.

An examination of the values of $A_e\lambda/vol$ for a series of airship models gives the following results:—

Airship.	R.29.	R.32.	R.33.	R.38.	R.80.
$A_e\lambda/vol$	5·67	5·7	5·5	5·5	5·65

and hence

$$\frac{A_e\lambda}{vol}\cdot\frac{(mY'_\beta)_h}{A_e}+\frac{(CN'_\beta)_h}{vol}$$

is approximately constant.

Thus for the purpose of finding a law of variation of R for different airships we may say that approximately $R\beta$ is proportional to $m\lambda/vol$, or since m is proportional to the volume of the ship, $R\beta/\lambda$ is constant, $=K$ say.

The following tables give the values of $R\beta/\lambda$ (β measured in radians) as calculated from model data and as observed in actual flight on a number of airships.

(i.) Calculated from Model Data.

	R.29.	R.29a.	R.29b.	R.33.	R.32.
λ feet.	1.70	1.83	1.80	2.5	2.03
β degrees.			$R\beta/\lambda$.		
1	0.925	0.87	0.88	0.88	0.87
2	0.925	0.87	0.88	0.95	0.88
3	0.91	0.85	0.86	0.93	0.87
4	0.90	0.84	0.85	0.93	0.87
5	0.88	0.82	0.83	—	0.85
6	0.87	0.82	0.83	0.93	0.83
8	0.87	0.81	0.82	0.91	0.83
10	0.86	0.80	0.81	0.88	0.81

(ii.) Full-Scale Observations.

Angle of rudder degs.	R.29.		R.29a.		R.29b.		R.33.		R.32.	
	β degs.	$R\beta/\lambda$.	β degs.	$R\beta/\lambda$.	β degs.	$R\beta/\lambda$.	β degs.	$R\beta/\lambda$.	β degs.	$R\beta/\lambda$.
5	1·75	0.925	2.6	0.96	3.6	0.89	4.6	0.98	3.03	0.89
10	2.9	0.92	4.0	0.95	5.1	0.89	5.5	0.93	4.19	0.89
15	3.9	0.91	4.9	0.93	6.3	0.91	6.7	0.94	5.22	0.93
									6.35	0.95

Mean value $R\beta=0.92\,\lambda$.

The agreement is good except perhaps in the case of R.29a, and the accuracy of the full-scale figures may be doubted in this case. Since R.29a is intermediate between R.29 and R.29b, the value of $R\beta/\lambda$ for R.29a could be expected to lie between the corresponding values for R.29 and R.29b, but this is not the case.

Mention might be made of the fact that although $R\beta/\lambda$ is approximately constant, there appears to be a general tendency for the value to decrease as β increases. The same tendency is observed in the results of the whirling arm experiments referred to. This effect may be taken into account if desired, but it has not been considered advisable with the data at present available to lay too much emphasis upon it.

8.5. The physical significance of $R\beta/\lambda=K$ is interesting

(*a*) When an airship turns in a circle of fixed radius, the angle of yaw will always be the same, no matter what the fin area may be, provided the C.P. of the forces on the fins is unaltered. The angle of rudders will change if the fin area be altered.

(*b*) For values of β of the magnitude usual in airships $R \sin \beta / \lambda = K$. Thus, if when a particular ship be turning in a circle about a certain centre, a perpendicular be drawn from the centre on to the axis of the ship, the foot of the perpendicular will always be approximately the same distance $K\lambda$ from the C.B. Further, the axis of the ship will, at this point, always be tangential to a circle concentric with that described by the C.B. so that $K\lambda$ defines a point at which the "local" angle of yaw* is zero, and the C.B. divides the distance between this point of zero yaw and the C.P. of the forces on the fins in a fixed ratio.

(*c*) Since the resultant speed of the ship is $r_0 R$, the side-slipping velocity $v_0 = -r_0 R \sin \beta$ and $r_0 v_0$ is approximately constant on any particular airship, or in general proportional to λ.

The Determination of Y_r

8.6. The method of determining Y_r on the complete ship is now evident, for in addition to the two equations (1) and (2) we have the additional relation $R\beta = K\lambda$.

The simplest method of procedure is to use this relation in conjunction with (1*a*) for determining graphically R and β for several rudder angles. The curves mY_β against β at various rudder angles are then utilised. From them the values of $mY_\beta = -mV(Y_r - u_0)/V$ are deduced at the appropriate values of β, whence knowing also R, mY_r is immediately obtained. Specimen curves applicable to R.29, R.29a and R.29b are given in Figs. 10, 10a and 10b. The lines joining the points defining the equilibrium positions are again approximately straight, indicating that Y_r does not vary appreciably with β.

The following values have been obtained as above for Y_r/u_0:—

Rudder angle.	R.29.	R.29a.	R.29b.
5°	0.74	0.60	0.53
10°	0.73	0.61	0.53
15°	0.72	0.62	0.53

Components Due to Hull and Due to Fins

8.7. In § 4 the assumption was made that $(N_\beta / Y_\beta)_f = (N_r / Y_r)_f$ or expressed otherwise that $(CN_r)_f = -\lambda (mY_r)_f$ and the agreement between the results based on this assumption and those obtained from full-scale observations apparently justifies it. We have then readily a means of estimating mY_r due to fins, by simply dividing by $-\lambda$, $(CN_r)_f$, which can be obtained experimentally (§7.3). In the three cases we have chosen as examples we find that $(Y_r/u_0)_f = 0.65$, 0.55 and 0.42, giving the mean value of 0.08 for $(Y_r/u_0)_h$.

Considerations similar to the above have been applied to R.32, R.33 and R.38; the agreement in all cases between the calculated results and those obtained from full-scale experiments is of the same order.

Roll and Pitch

8.8. It is to be remembered that an airship in circling flight rolls through a small angle since the C.G. is below the C.B. The angle of roll can be calculated by the ordinary methods of dynamics. In the calculations described in § 8.2 the models were not rolled; the effect on the forces measured in the wind channel is believed to be small, from the symmetry of the airship form. From the close-

* Local angle of yaw. It is obvious that in circling motion the angle between the axis of the airship and the direction of motion of any point on the axis is different for different points. The angle is here termed local angle of yaw to distinguish it from the true angle of yaw at the C.B.

ness of the agreement between model and full-scale it is concluded that in discussing airship motion it will be sufficiently accurate to neglect roll.

Further, the model experiments were conducted on unpitched models, whereas in the full-scale trials the airship was almost invariably pitched. The effect of a slight pitch on the lateral motion is then believed to be small.

Stability of Equilibrium

8.9. Before dealing with the general mathematical theory of stability it would be as well to consider at this juncture the information to be derived from the curves used in connection with the study of the equilibrium conditions.

An examination of Figs. 9 and 9b will show that at the origin the nature of the curves is different in the two cases. The curve giving $R\beta$ constant is, in Fig. 9, steeper than the curve $-N_\beta/VN_r$ with $\alpha=0$, whereas the reverse is the case in Fig. 9b, and the two curves intersect at points other than that defined by $\beta=0$, viz., at $\beta=\pm 1.5$ degrees approximately. Thus, in this latter case there are three positions of equilibrium for the airship with rudders amidships.

In the case of R.29 $-\partial/\partial\beta\,(N_\beta/VN_r)<\partial/\partial\beta\,(1/R)$ when $\beta=0$ (Fig. 9) and in the case of R.29b $-\partial/\partial\beta\,(N_\beta/VN_r)>\partial/\partial\beta\,(1/R)$ when $\beta=0$ (Fig. 9b). It was stated in § 7.4 that when $\beta=0$ $\partial/\partial\beta\,(CN_r)=0$, and since

$$R\beta=K\lambda$$

$$\frac{\partial}{\partial\beta}\left(\frac{1}{R}\right)=\frac{1}{K\lambda}$$

hence these two inequalities become

$$-\frac{1}{VN_r}\frac{\partial}{\partial\beta}(CN_\beta)<\frac{1}{K\lambda} \quad \text{R.29} \quad . \quad . \quad . \quad . \quad (6)$$

and

$$-\frac{1}{VN_r}\frac{\partial}{\partial\beta}(CN_\beta)>\frac{1}{K\lambda} \quad \text{R.29b} \quad . \quad . \quad . \quad . \quad (6a)$$

Now imagine plotted on a β base the total moment $CN_\beta+V/R\,.\,CN_r$. By symmetry this vanishes when $\beta=0$. If the airship receive a small displacement in the direction β positive and the moment is positive in this new position, then it will tend to displace it further and equilibrium is unstable at $\beta=0$.

Thus if

$$\frac{\partial}{\partial\beta}(CN_\beta)+\frac{\partial}{\partial\beta}\left(\frac{V}{R}\,.\,CN_r\right)>0$$

or

$$\frac{\partial}{\partial\beta}(CN_\beta)>-\frac{V}{K\lambda}\,.\,CN_r$$

then equilibrium is unstable at $\beta=0$.

This condition is identical with (6a) above and may be written in the form

$$-N_v>-\frac{N_r}{K\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

since if β be measured in radians

$$VN_v=-\frac{\partial}{\partial\beta}(N_\beta) \text{ at } \beta=0 \text{ (see § 11.3, equation (7)).}$$

Similarly equilibrium will be stable if

$$-N_v<-\frac{N_r}{K\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence equilibrium will be stable or unstable according as to whether

$N_r/N_v >$ or $< K\lambda$ or $R\beta$, *i.e.*, according as to whether the line $R\beta$ constant is steeper or less steep than the curve of $-N_\beta/VN_r$ with rudders amidships at $\beta = 0$. Thus R.29 is stable and R.29b unstable in rectilinear flight. The stable motion of the latter with rudders amidships is circling motion, and the calculated radius of the circle agrees with that observed on full-scale. Similar considerations applied to the lateral force curve furnish another condition of stability, viz.,

$$(Y_r - u_0)/Y_v < K\lambda \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Combining conditions (8) and (9) we obtain

$$Y_v N_r > N_v (Y_r - u_0) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

We have thus derived a stability criterion which can be written in one of three forms. It would appear that the most convenient form for use is

$$N_r/N_v > K\lambda$$

since the only model data needed are yawing moments in translational motion and N_r; λ can be approximately taken to be $0.9\,\lambda_1$ where λ_1 is the distance of the centre of area of the fins from the C.B.

8.10. Let us consider further the curves of Fig. 9b applicable to R.29b, an airship which we have seen to be laterally unstable in rectilinear flight. Imagine a curve corresponding to a very small positive rudder angle α_0, *i.e.*, the curve which, at a value of β (β_0) intermediate between 0 and 1.5°, would be tangential to the line $R\beta = K\lambda$ (see Fig. 11). This curve would cut the latter line at a negative value of β numerically greater than 1.5°. This point of intersection represents the normal stable equilibrium attitude, the airship turning to port with the rudders to port. At $\beta = \beta_0$ equilibrium will be neutral.

Again, imagine a curve corresponding to a rudder angle α_1 such that $0 < \alpha_1 < \alpha_0$. This curve will cut $R\beta = K\lambda$ in three points, two at which β is positive, $0 < \beta < \beta_0$ and $\beta_0 < \beta < 1.5°$, and one at which β is negative and such that $|\beta_1| > |\beta| > 1.5°$. This negative value of β represents again a normal attitude of stable equilibrium. The value of β intermediate between zero and β_0 corresponds to a position of unstable equilibrium, and it is obvious that there is no position of stable equilibrium at angles of yaw numerically less than β_0 and that consequently the airship under consideration would not fly uncontrolled in a circle of radius greater than $R = K\lambda/\beta_0$.

The third value of β, *i.e.*, that intermediate between β_0 and 1.5°, defines an attitude of stable equilibrium and represents the rather peculiar possibility of the airship turning to starboard with the rudders to port. Thus, in order to turn in a circle whose radius is intermediate between $K\lambda/\beta_0$ and $180/\pi \cdot K\lambda/1.5$, the rudders must be inclined in the direction opposite to the normal. The usual range within which β varies then is divided into three well-defined regions.

(i.) $\beta > 1.5°$ numerically. Airship stable, normal control.
(ii.) $\beta_0 < \beta < 1.5°$ numerically. Airship stable, reversed control.
(iii.) $0 < \beta < \beta_0$ numerically. Airship unstable.

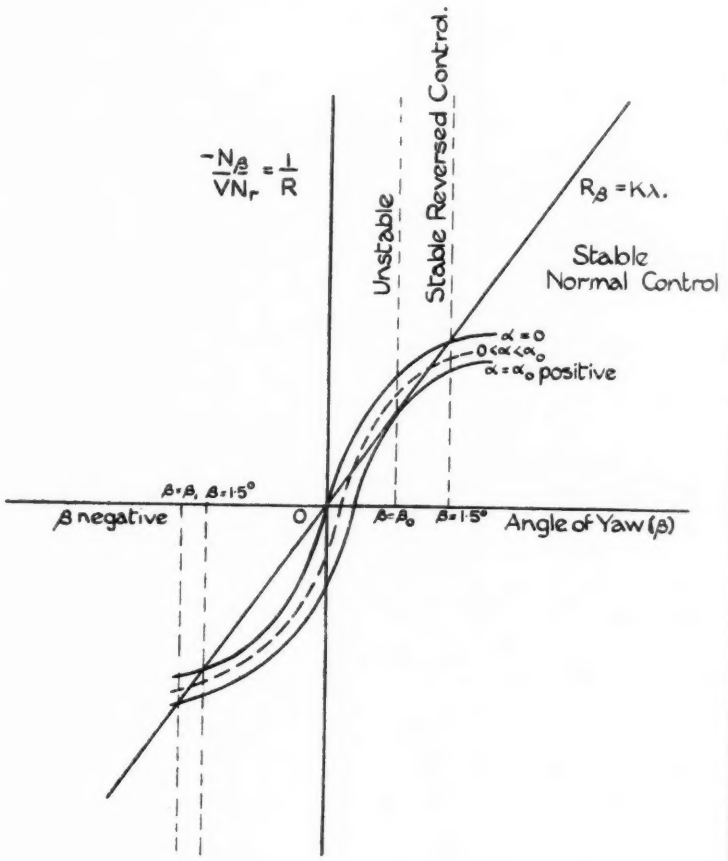
It is not known whether this phenomenon of reversed control has ever been found to exist in practice; the motion which is theoretically possible may be impossible under the disturbed conditions of actual flight.

IX. *The Distribution of Pressure Over an Airship Model*

9.0. As stated in § 1.0, it is probable that the data of the most direct importance for calculation of the strength of the structure of an airship are those relating to the distribution of aerodynamic pressure over the hull. Data of this nature are also of great use in non-rigid design in calculating the nose stiffening required to enable the fabric to withstand the high pressure over the forward end at high speeds.

Fig. 11.

An exaggerated sketch of Fig 9b near $\beta = 0$, illustrating § 8·10.



Experimental evidence of the pressure distribution over models is available for one or two types, and though the data do not cover such a wide range of models as in the case of the data already considered, yet they suffice to show the general variation that may be expected. Information regarding the effects of rotation is unfortunately lacking to an even greater extent in the case of pressure determination than in the case of total forces and moments. Methods of estimating them in certain cases are suggested, but confirmation of the results is still needed. We shall direct attention, first of all, to purely translational motion. As a typical example the results of recent experiments on a model of R.33 will be examined.*

Method of Experiment

9.1. A number of tubes are inserted in the model flush with the surface and extending along the entire length of the model. One end of each tube is sealed, the other end connected to one side of a Chattock manometer by means of suitable connections. A series of holes are made in each tube, one hole only at a time being left open in any tube. The pressure at this hole is observed and then the hole is sealed and another opened, and so on. The model is so supported in the channel that it can be yawed or pitched so that pressures can be observed under circumstances similar to those under which the forces and moments due to translation are measured.

If the model be a body of revolution with no fins or cars attached, and mounted so that it can be rotated about its own axis, one tube only will suffice.

The other side of the Chattock manometer is connected to a hole in the side of the channel and consequently the pressure observed is the difference between the normal pressure on the model at any point and the pressure at the hole in the side of the channel. Since the pressure required is the difference between the normal pressure at the hole in the model and the static pressure in the channel at that point, a correction equal to the difference between the static pressure at the side of the model and that at the hole in the side of the channel must be applied. This correction varies along the channel (see § 4.0).

Pressures at Zero Yaw. Form Resistance

9.2. Fig. 12 shows the pressures along the model (expressed in terms of $\rho V^2/2$) when the axis of the model is parallel to the wind direction.

Let the pressure at any point P be denoted by p, and let the plane XOP make an angle ϕ with $XO\bar{Z}$ (see § 2.0). Also let r be the radius of the cross section through P.

If the angle of incidence be zero, and the model be a body of revolution without fins or cars, p is independent of ϕ and it is easy to show that the drag due to normal pressures

$$mX = 2\pi \int_{O_1}^{O_2} p \,.\, r \,.\, dr$$

$$= \int_{O_1}^{O_2} p \,.\, da$$

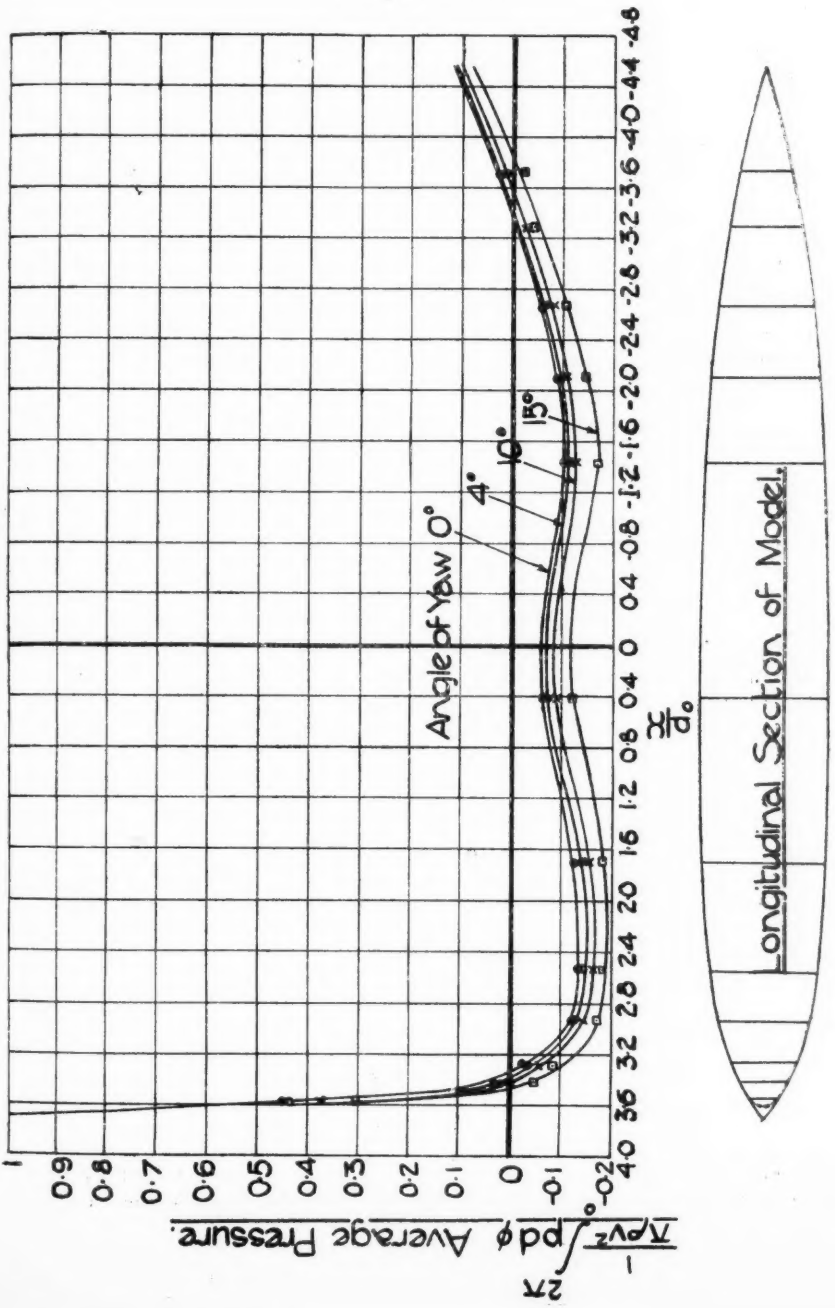
where a is the area of the cross section (πr^2) and O_1 and O_2 the zero values of r and a at the head and tail respectively.

* R. & M. 801. Experiments on other types of airship hulls are described in R. & M. 107, 246, 600.

FIG. 12.

PRESSURE DISTRIBUTION ON THE HULL OF R.33.

Average Pressures.



On calculating the value of this integral from the curve given in Fig. 12 (zero yaw) it is found that the drag or form resistance is zero* and we therefore conclude that the resistance of the model is due to skin friction or tangential force. This has been found to be so in the case of another model tested, but the form resistance of two models of an older type appeared to be about 5 per cent. of the total resistance. Moreover, in the case of a prolate spheroid 4 : 1, the ratio of drag due to the normal pressures may be about a third of the total drag, so that it is probable that the above conclusion is not generally true, even for models of approximately streamline form. There is apparently some scope for further research in this direction. It is easy to show that if the skin friction coefficient is constant over the surface of an airship model then the drag coefficient due to friction will be proportional to the area of a longitudinal section through the axis.

Again, if the form resistance were always zero, the skin friction coefficient must depend on the form of the body, since it has been found that the total drag of an airship model is not proportional to the area of axial section. The deviation from proportionality is not very marked and it is just *possible* that a small allowance for form resistance would be sufficient to explain this deviation. At present, however, no definite conclusions can be drawn.

Pressures with the Axis of the Model Inclined to the Wind Direction

9.3. (*a*) *Average Pressures. Longitudinal Force.*—It is obvious that if the angle of incidence be not zero, the pressure at P will depend on the angle ϕ and the longitudinal force will be

$$\int_{s_1}^{s_2} \int_{0}^{2\pi} p d\phi \,.\, r \cos\theta \,.\, ds$$

where ds is the element of length along an axial section through P, and s_1 s_2 the values of s at the head and tail respectively, $s = 0$ in the plane XOY. Also θ is the angle between the normal at P and OX.

The average pressure at the section $x =$ constant through P is

$$\frac{1}{2\pi} \int_{0}^{2\pi} p d\phi = P_1$$

This quantity is shown also in Fig. 12 for the model chosen as an example at various angles of yaw. The longitudinal force may then be written as

$$2\pi \int_{s_1}^{s_2} P_1 r \cos\theta \,.\, ds$$

$$= 2\pi \int_{O_1}^{O_2} P_1 r dr \text{ or } \pi \int_{O_1}^{O_2} P_1 d(r^2)$$

An evaluation of this integral from the quantities represented in Fig. 12 shows that in the present case the longitudinal force is also zero for angles of yaw

* See § 4.8.

other than zero, and it is therefore concluded that, on this particular model, not only is the longitudinal force at zero yaw due to tangential forces or skin friction, but that the same result holds at other angles of incidence.

(*b*) *Resultant Pressures. Lateral Force and Yawing Moment.*—In order to derive the lateral force and yawing moment we must determine the component in the plane XOY of the resultant pressure per unit run at any section.

The component of p parallel to the plane XOY is $p \sin \phi$. Hence the component in the plane XOY of the resultant pressure on the section through P

$$\int_0^{2\pi} p \sin \phi \, . \, r d\phi = \int_{-z_0}^{z_0} p \, . \, dz = P_2$$

where z_0 is the maximum value of z at the section through P ($z_0 = r$).

The resultant lateral force is

$$\int_{s_1}^{s_2} P_2 \sin \theta \, . \, ds = \int_{x_1}^{x_2} P_2 \, . \, dx$$

and the yawing moment is

$$\int_{x_1}^{x_2} P_2 x \, . \, dx$$

x_1 and x_2 are the distances of the head and tail respectively from the axis about which moments are taken, in this case from the C.B.

Fig. 13 shows the variation of P_2 along the axis of the model at different angles of yaw as well as the integral

$$\int_{x_1}^{x_2} P_2 \, . \, dx$$

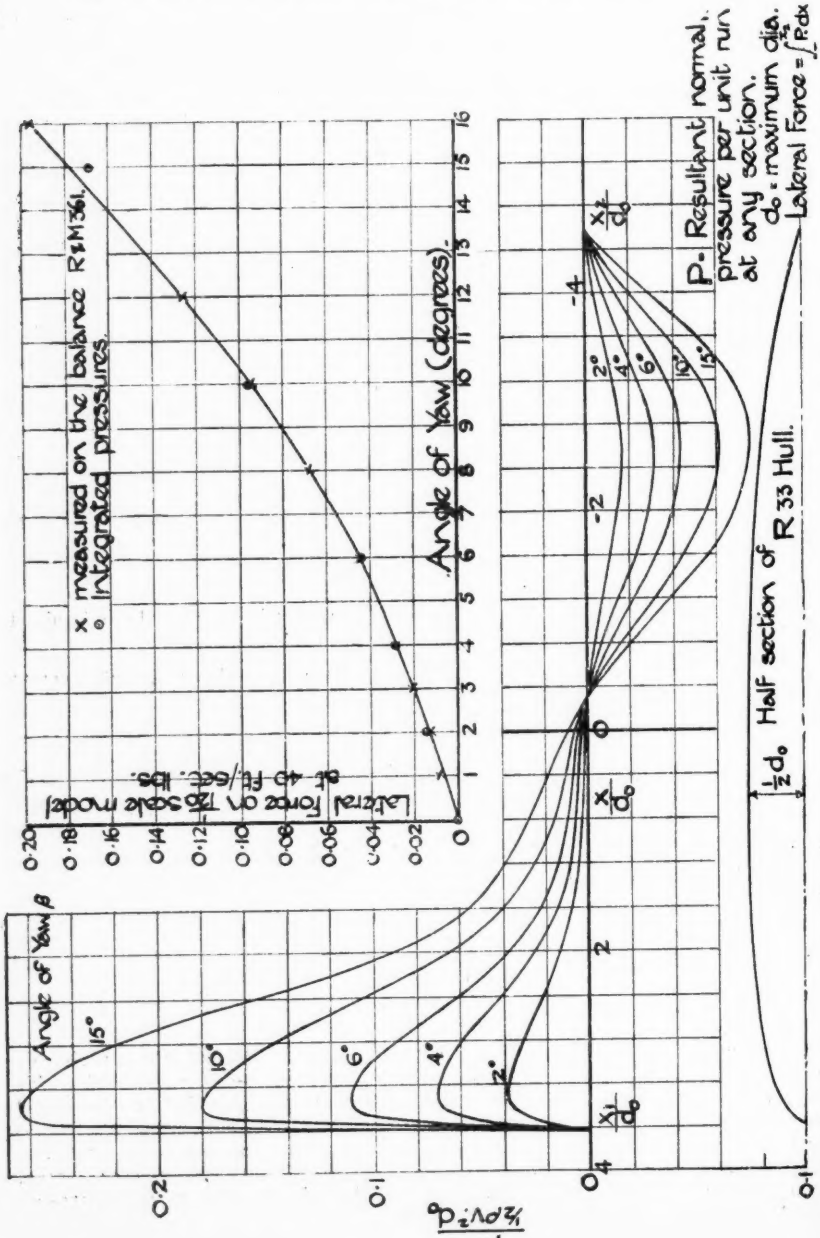
Reference to the figure will show at once that the integrated lateral force agrees remarkably well with the force measured on the aerodynamic balance. The same remark also applies to

$$\int_{x_1}^{x_2} P_2 x \, . \, dx$$

It would appear then that the lateral force and yawing moment are entirely due to normal pressures in the case of the model considered. It is probable that this result is not universally true, for instances are known in which the measured lateral force is appreciably greater than the integrated. In the case of a spheroid the former is 1.1 times the latter. It is possible that the equality of the two quantities is due to the fact that the model hull of R.33, chosen as an example, has a lower lateral force per unit elevation area than the majority of airship models (see Appendix I., Table 1).

FIG. 13.

DISTRIBUTION OF RESULTANT PRESSURES OVER THE HULL OF R.33.



Speed Effect

9.4. There does not appear to be any consistent variation of the pressure coefficient $2p/\rho V^2$ with speed, except perhaps at the after end of the model. The available evidence tends to indicate that the coefficient decreases as the speed increases, but examples of the opposite effect are not unknown.

Effect of Cars

9.5. It is difficult to draw any general conclusions regarding the effect of cars; the disposition and arrangement of cars vary so much from ship to ship that results on one model may not be applicable to another. It appears probable, however, that except in their immediate vicinity cars have no appreciable effect upon the hull pressures. It is certain that the effect on the integrated pressures would be negligible.

Effect of Fins

9.6. The same remarks apply to fin pressures. Their effect may of course be quite considerable at the after end between a horizontal and vertical fin for example.

Pressures on the Fins

9.7. So far as is known, data regarding the pressures on the fins have been obtained on two models only. Some observations were made on the 1/120th scale model of R.33, but the main object of these measurements was to enable a general check to be obtained on the results of experiments conducted on a large model of R.32, scale 1 : 44.* Moreover, some experiments have been conducted on the actual airship in the case of R.32 and a less complete set of results is available on R.38. Unfortunately, some of the observations that had been taken on the latter were lost at the time of the disaster.

A predominant feature of the pressures on a vertical fin is the rapidity with which the magnitude of the pressures increases with angle of yaw.

The results of the full-scale experiments revealed the fact that at certain points on the fins, particularly near the leading edge, the pressures were unexpectedly high; although the model results do not show the presence of pressures of the same magnitude, yet the general nature of the results tends to confirm the full-scale results. It is possible that the angle of yaw at the fins of the actual airship was greater than it was supposed to be and that the high pressures observed may be attributed to this cause.

It should be mentioned that the pressures observed on the full-scale fin when the airship was turning with the rudders locked are in satisfactory agreement with values predicted from the corresponding model experiments. For this prediction the angle of yaw of the model fin was assumed to be the mean local angle of yaw at the fin of the actual airship.

Comparison with Theory

9.8. In a paper by Fuhrmann† an account is given of experiments conducted to determine the distribution of pressure on a series of models of approximately streamline form. The shapes of these models were such that the pressures could be calculated by the usual methods of pure hydrodynamics. A consideration of the flow of a uniform stream in motion past a system of sources and sinks formed the basis for the determination of the shapes of the models. The calcula-

* R. & M. 868.

† Jahrbuch der Motorluftschiff Studiengesellschaft 1911-1913.

tions involved in this investigation are rather heavy and the method is applied to cases where the axis of the model is parallel to the direction of motion. The results of theoretical calculations agreed very well with the experimental pressures at zero yaw, except of course near the tail, and indicated the desirability of extending the comparison to yawed models. An investigation on the above lines into the theoretical pressures on a model whose axis is inclined to the direction of motion is probably impossible, or if not so, it would involve very laborious calculations. Accordingly, it appeared necessary to choose a shape to which hydrodynamical methods could be applied with greater ease. Now the head of an airship approximates to a prolate spheroid ratio of axes about 4 to 1, and thus a spheroid is evidently a suitable model. A comparison of the pressures on a spheroid determined experimentally and theoretically* shows that theory and experiment are in close agreement for three-quarters of the length of the model, and even at angles of yaw of 20 degrees on the windward side. The agreement is naturally better on the windward side, but even on the leeward side it is good for one half the length of the model. Moreover, the pressures over the forward half of the head of an airship hull (provided the head is ellipsoidal) agree well with those calculated theoretically on the spheroid, indicating that for the purposes of nose stiffening investigations the theoretical calculations conducted on a spheroid would eliminate the necessity for experiments. Further, if the head be not exactly spheroidal it will be sufficiently accurate for these purposes to determine the spheroid which most closely fits the extreme forward end, and calculate the pressures upon that part of it which coincides with the airship head.

Pressure Distribution on the Hull in Curvilinear Flight

9.9. As far as is at present known to the author, no experimental evidence is available regarding the distribution of pressure in curvilinear flight. Experiments are in contemplation in which the model will be mounted on a whirling arm. It is obvious, however, that before adequate stress calculations can be undertaken on a proposed airship, some indication of the probable pressures are essential. The following method of estimating the resultant lateral pressures suggested itself, but did not yield results which could be believed.

When an airship turns in a circle the angle of yaw varies along the ship, in other words, the angle between the axis of the ship and the direction of motion at any point on the axis varies along the axis. Thus, at the point $x = R \sin \beta$ we have seen that the angle of yaw is zero, whereas at the tail the angle of yaw is approximately twice the real angle of yaw, *i.e.*, the local angle of yaw at the C.G. Now knowing the attitude of equilibrium in curvilinear flight from calculations described in Section VIII., the local angle of yaw at any point can be calculated. Further, if the resultant lateral pressures at various angles of yaw be determined experimentally for all sections along the model, a strip theory can be applied to plot the resultant lateral pressures in curvilinear flight. Plotting in one case against x^1 and in the other against x^2 and determining the areas of the curves, give the total lateral force mY_0 and total yawing moment CN_0 respectively on the hull. Knowing the lateral force and yawing moment due to translation mY_β and CN_β, it is easy to find Y_r and N_r. In the example worked out (see Fig. 14) this method resulted in a large negative value of Y_r and a value of N_r about twice that observed in the wind channel. Moreover, the value of mY_0 obtained was such that, if correct, the airship would fly in a circle with r the velocity of the side-slip and r the angular velocity of the same sign. This, however, is impossible.

In spite of the failure of the method, it suggests another by means of which a distribution of resultant pressure which gives the correct results may be derived.

* R. & M. 600. A more complete investigation is in progress.

† x is the distance of any section from the C.B.

Thus, knowing the total lateral force and yawing moment corresponding to a particular value of β, a curve of resultant lateral pressures may be plotted against x, such that its area gives the appropriate lateral force, and further when the ordinates are plotted against x^2 the appropriate yawing moment is derived. There may, however, be an infinite number of such curves, and a method of deciding on the most probable curve is necessary. If it is assumed that theory and experiment agree to the same degree of approximation in curvilinear motion as in translational motion then the method is supplied. It is possible to calculate the pressure distribution over a spheroid supposed moving in a circle, and hence also the resultant lateral pressures. Using results obtained in this manner as a guide in drawing the curves near the nose of the airship model, we obtain a curve which cannot be widely in error, provided the above assumption is justified. Examples of curves obtained with and without reference to theory are shown in Fig. 14. More details of the calculations are given in Appendix II.

9.10. It is now evident that a method which enables the equilibrium attitudes of an airship to be determined from model data is essential for an investigation into the strength of structure based upon pressure distribution. For, before the resultant pressures in curvilinear flight can be estimated along lines similar to those described above, the total forces acting on the airship and the general motion must be known. Moreover, supposing that an experimental determination of the pressure distribution be undertaken on a model mounted on the whirling arm, the relation $v_0/r_0 = -K\lambda$ between velocity of sideslip and angular velocity, must first be assumed before the results can be applied to an airship in curvilinear flight.

X. *Acceleration Effects. Virtual Mass Inertia Coefficients*

10.0. It is a well-known fact in hydrodynamics, when considering the accelerated rectilinear motion of a body in an ideal fluid, that the fluid exerts a force equal to $KM\,dV/dt$ on the body, where M is the mass of the fluid displaced by the body, V the velocity at any instant and K a coefficient, generally termed inertia coefficient, depending on the form of a body. In the cases of a sphere and a circular cylinder moving parallel to itself K is 1.5 and 2 respectively. The effect of acceleration thus came to be regarded as caused by a quantity of fluid being dragged along by the body and requiring an additional force proportional to its mass to propel it.

In a real fluid the resistance of an accelerated body can be shown by dimensional theory* (*cf.* § 3.1) to be expressible in the form

$$R = \rho V^2 L^2 f\left(\frac{VL}{\nu}, \frac{\dot{V}L}{V^2}\right)$$

or

$$R = \rho V^2 L^2 f_1\left(\frac{VL}{\nu}\right) + \rho \dot{V} L^3 f_2\left(\frac{VL}{\nu}\right) + , \text{ etc.},$$

so that the coefficient $\rho L^3 f_2\,(VL/\nu)$ corresponds to the added mass referred to above. It would appear from this expression that the conception of virtual mass is only valid when the acceleration is small and terms of higher order than the first in $\dot{V}$ can be ignored.

The inertia coefficients of a prolate ellipsoid in an ideal fluid† have been calculated for head-on motion, lateral motion parallel to a minor axis and purely rotational motion about a minor axis. In the case of head-on motion the coefficients vary from 1.5 for the sphere to 1.02 for an ellipsoid of fineness ratio 10:1. For the lateral motion, the corresponding variation is from 1.5 to 1.96 and for rotational motion from 0 to 1.88.

* R. & M. 612. Effects of Accelerations on the Resistance of a Body.
† R. & M. 623. Lamb.

A Pressures in Rectilinear Motion, Yaw 7·3°.
B ,, ,, Curvilinear ,, assuming variable yaw gives $(Y_r)_h = -0.63\,V$.
C ,, ,, ,, ,, giving a correct total force and moment $(Y_r)_h = 0.09\,V$.
D ,, ,, ,, ,, using theoretical curve as a guide.

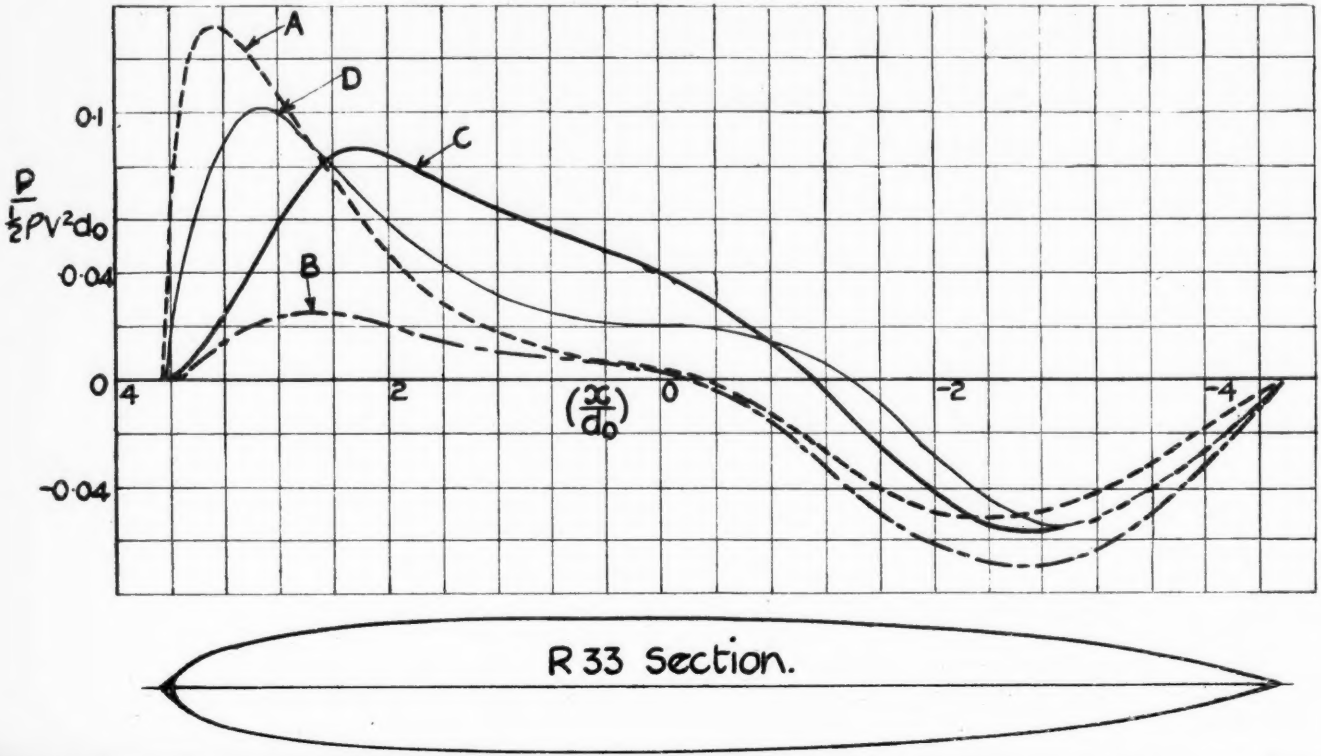


FIG. 14.
RESULTANT PRESSURES ON THE HULL OF R.33 IN CURVILINEAR MOTION.

10.1. An attempt at determining the virtual mass effect experimentally* has been conducted by swinging small models one inch in diameter in air and water. In the latter case the speed of the water current varied from zero to 1.6 inches per second. The method depends on the assumption that the virtual mass in air may be neglected in comparison with the mass of the body. If m be the mass of the body, m_v the additional mass, w_w and w_a the weight of the body in water and air respectively, and T_w and T_a the periods of the oscillations in water and air respectively, then

$$\frac{m + m_v}{m} = \left(\frac{T_w}{T_a}\right)^2 \frac{w_w}{w_a}$$

is the inertia coefficient.

The method has obvious drawbacks, the chief of which is the fact that the body in oscillating is always moving through fluid which has been disturbed. To check this, two spheres equal in diameter but of different weights were compared; the results gave values of m_v equal to 83 and 83.5 per cent. of the mass of water displaced. This value of 83 per cent. is considerably in excess of the 50 per cent. for a sphere in ideal fluid, and this is consistent with the physical idea that viscosity would cause a greater mass of fluid to be involved in the acceleration effect.

The results obtained with a series of models made up of a cylindrical centre portion and hemispherical ends, and also with ends roughly of streamline form are shown in Fig. 15; in addition the theoretical results on a prolate spheroid are given. It will be seen that though the forms of the curves are somewhat similar, the experimental values are in all cases higher. This may be due to one or more of several reasons, *e.g.*, viscosity effect, the higher resistance coefficient of the experimental models. Mention may be made of results of similar experiments obtained on an airship model about 3 feet long and 5.6 inches in diameter. The experiments were conducted in the William Froude tank at speeds up to 300 ft./min., and gave values of 12 per cent. and 100 per cent. for the virtual mass effect in longitudinal and lateral† motion respectively. Attention might be drawn to the results quoted at the conclusion of Section IV., § 4.9. It was seen there that an allowance of about 15 per cent. for inertia effects in longitudinal motion would bring the resistance of an airship as predicted by the second method of § 4.9 into close agreement with that observed by deceleration trials. Other experimental methods which have been applied to longitudinal motion are described in two reports, R. and M. 612 and 590.

10.2. When the problem of airship stability was first considered the acceleration effect was introduced by imagining the mass and moment of inertia of the airship to be increased. Thus, for example, the mass used in the x equation was the true mass plus the added mass in longitudinal motion; the mass used in the y equation was the true mass plus the added mass in lateral motion, and so on. A more logical way of introducing the effect is to consider it purely as an acceleration effect without reference to mass. Thus instead of assuming the lateral force mY to be equal to

$$m\,(Y_0 + uY_u + \ldots + rY_r)$$

additional terms

$$m\,(\dot{u}Y_{\dot{u}} + \qquad + \dot{r}Y_{\dot{r}})$$

are introduced.

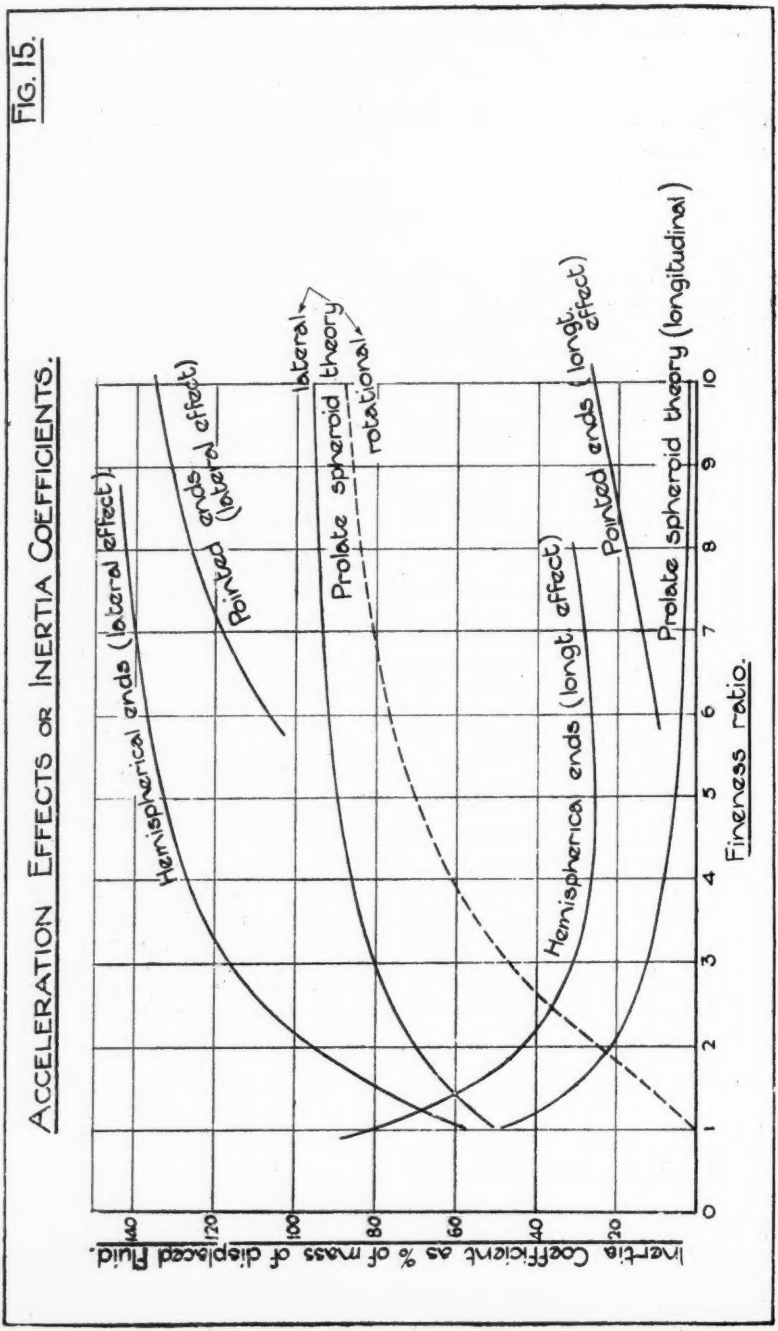
In this expression

$$Y_{\dot{v}} = 1 - \frac{m + m_v}{m}$$

in lateral motion.

* R. & M. 613.

† The model was moving forward, *i.e.*, parallel to its axis in both longitudinal and lateral motions. In the first case it oscillated about an axis parallel to a transverse axis, and in the second about an axis parallel to the longitudinal axis.


ACCELERATION EFFECTS OR INERTIA COEFFICIENTS.
FIG. 15.
Inertia Coefficient as % of mass of displaced Fluid.
140
120
100
80
60
40
20
0
1
2
3
4
5
6
7
8
9
10
Fineness ratio.
Hemispherical ends (lateral effect)
Pointed ends (lateral effect)
Prolate spheroid theory
lateral
rotational
Hemispherical ends (longit. effect)
Pointed ends (longit. effect)
Prolate spheroid theory (longitudinal)

The experiments mentioned above deal only with such symbols as $Y_{\dot{v}}$, *i.e.*, the derivatives in which the force component along a certain axis is associated with the acceleration component along the same axis and similarly for couples. By symmetry these are the only acceleration factors that enter into the problem when the motion is rectilinear. The question naturally arises, whether or not in curvilinear flight the derivatives such as $Y_{\dot{r}}$ exist, and if so how far they affect the solution of the equations of motion?

No experimental means of ascertaining this is available. An attempt, however, was made to examine the acceleration effects on a prolate spheroid in curvilinear motion. It was found that the force and couple relative to one axis associated with linear or angular velocities relative to another are zero, and this was therefore assumed to be the case for the bare hull. The effect of fins was considered by imagining the fin to be a circular disc placed at the after end of the spheroid. The only derivatives considered were

$$X,\ Y,\ N_{\dot{u}},\ _{\dot{v}},\ _{\dot{r}}.$$

and it was found that the fins contributed to the values of $Y_{\dot{v}}$, $Y_{\dot{r}}$, $N_{\dot{v}}$ and $N_{\dot{r}}$.

The manner in which these quantities enter into the stability calculations will be seen in Appendix III.*c*. To a very close approximation $Y_{\dot{r}}$ and $N_{\dot{v}}$ may be ignored.

Reliable information on this section of airship data is thus seen to be exceedingly scanty. It is, however, possible that full-scale performance could be of great assistance if facilities for the appropriate experiments were available. The method of utilising such experiments will be mentioned in § 13.0 and Appendix IV.

XI. *The Mathematical Theory of Stability*

11.0. The mathematical theory of airship stability follows closely the lines of aeroplane stability for the study of which the theory was developed by Bryan. It has been fully described and discussed by several writers elsewhere, so that a detailed consideration of the fundamental principles would be superfluous at present. The application of the results of experiments on models, both to aeroplanes and airships, was originated by Bairstow, by whom, in collaboration with others, the theory was extended to cover the most general motion of an aeroplane. So far as the airship is concerned, the problem dealt with was that of rectilinear motion.* This was later extended by Nayler and the writer† to include curvilinear motion.

In the case of rectilinear flight longitudinal motion (*i.e.*, motion in the plane of symmetry) and lateral motion (*i.e.*, motion in a horizontal plane), including rolling, are separable and the analysis of the motion depends on two bi-quadratics, one applied to each type of motion. The calculations involved are by no means heavy under these circumstances and fairly simple criteria for stability can be derived.

The solution of the general case of curvilinear motion even in a horizontal plane does, however, involve exceedingly laborious and tedious calculations. The work is so lengthy that it is unlikely to be of any great use to the designer. As in the case of the aeroplane, the determination of the motion of an airship depends on the solution of an octic.

Now we have seen in § 8.8 that banking and pitching do not appear to affect steady motion in a horizontal plane to any serious extent, thus it would seem justifiable to ignore in the general equations all terms involving roll and pitch.

* R. & M. 257.

† R. & M. 602. See Appendix III. *a* and *b*, in which it should be noted that the system of axes is different from that described in § 2.0.

The octic equation thus reduces to a cubic equation* and is comparable with the equations obtained by Crocco† for lateral stability or directional stability, as it should now perhaps be called since roll is neglected. Bryan severely criticises Crocco's method as applied to aeroplanes, but there seems no reason to doubt the applicability of the methods to airships if roll has as little effect in slightly disturbed motion as it has in steady curvilinear motion. It is obvious, however, that such methods cannot hold good in the case of aeroplane stability.

Moreover, so many of the quantities that enter into the coefficients of the octic equations have to be estimated, that it is probable that no accuracy is lost by adopting the simpler method of considering the stability of an airship in motion in a horizontal plane. In addition, the stability criterion which can be deduced from the cubic equation is of the same form as the criterion derived from the biquadratic of rectilinear motion. Thus it becomes possible to compare more closely the degree of stability possessed by a certain airship when flying in a straight and in a circular path.

An outline of applications of the theory of small oscillations to the problem of stability will be given in Appendices III.*a*, III.*b* and III.*c*, attention being confined for the present to the more general conclusions, without reference to acceleration effects. In the case of rectilinear motion the condition of lateral stability resulting from a solution of the equations of motion is exactly the same as that derived from the consideration of the equilibrium curves in § 8.9, viz.,

$$Y_v N_r > N_v (Y_r - u_o) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

A similar criterion for stability is obtained for longitudinal motion, viz.,

$$Z_w M_q > M_w (Z_q + u_o) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The biquadratic for longitudinal motion may be written in the form

$$(\lambda - X_u)\left\{(\lambda - Z_w)\left(\lambda^2 - \lambda M_q + \frac{Fh}{B}\right) - \lambda M_w (Z_q + u_o)\right\} = 0 \quad . \quad . \quad (3)$$

where F is the buoyancy of the airship and h the distance of the C.B. above the C.G. The resistance derivatives are referred to axes through the C.G. (see § 2.0). The significance of λ in this equation may be explained as follows. The equation is derived from the usual dynamical equations of motion with reference to moving axes (Hayward's equations) by assuming the various velocity components proportional to $e^{\lambda t}$, for example, $\dot{u} = \lambda u$, and writing the force components $X = X_o + uX_u +$, etc., as in § 2.2. On eliminating the velocity components u, etc., the biquadratic of which equation (3) is a simplification, follows. The nature of the oscillations of the airship depends clearly upon the values of the roots of this equation. If λ be real and positive, a disturbance of the form $u = ae^{\lambda t}$ will increase with time and the motion is unstable. A real negative value of λ, on the other hand, corresponds to stable motion, since the disturbance will then subside and $-\lambda$ will be the coefficient of subsidence. Complex values of λ are associated with oscillatory motion, the oscillations being stable or unstable according as the real part of λ is negative or positive as in the case of real roots, and the imaginary part of λ gives the periods of the oscillations.

One root of equation (3) above is X_u and the loss or gain of horizontal speed is to this degree of approximation independent of pitching or of vertical motion of the C.G.

Since for stability the real part of λ must be negative, the condition of longitudinal stability can be derived from the second factor of the above equation from a consideration of changes of sign (Appendix III.*a*, R. and M. 257, p. 19).

* See Appendix III.*c*.

† Bollettino della Societa Aeronautica Italiano. See A.C.A. Reports, 1909-10, pp. 41, 161.

11.2. In a similar manner the lateral biquadratic factorises approximately, thus

$$\left(\lambda^2 - \lambda L_p + \frac{Fh}{A}\right)\left\{\lambda^2 - (Y_v + N_r)\lambda + \begin{vmatrix} Y_v, & Y_r - u_0 \\ N_v, & N_r \end{vmatrix}\right\} = 0 \quad . \qquad (4)$$

The first term represents a damped oscillation in roll, which is independent of side-slipping and yawing, and the condition (1) for lateral stability follows from the quadratic factor (Appendix III.*a*, R. and M. 257, p. 19).

11.3. For curvilinear motion in a horizontal plane the cubic equation derived by neglecting pitch and roll from the general equations, under certain conditions* approximately factorises thus—

$$(\lambda - X_u)\left\{\lambda^2 - \lambda(X_r + Y_v) + \begin{vmatrix} Y_v, & Y_r - u_0 \\ N_v, & N_r \end{vmatrix}\right\} = 0 \quad . \quad . \qquad (5)$$

which is merely the first factor of equation (3) and the second factor of equation (4). It must be remembered, however, that even for the same airship the derivatives are not in this case equal to those of § 11.2 and § 11.3. The airship is now turning and is yawed, hence the values of the derivatives must be taken at the appropriate angle of yaw for equilibrium. At an angle of yaw β the following are the values of Y_u and Y_v:—

$$Y_u = -\frac{1}{V}\left\{-2Y_\beta \cos\beta + \sin\beta\,\frac{\partial Y_\beta}{\partial\beta}\right\} \quad . \quad . \quad . \qquad (6)$$

$$Y_v = -\frac{1}{V}\left\{2Y_\beta \sin\beta + \cos\beta\,\frac{\partial Y_\beta}{\partial\beta}\right\} \quad . \quad . \quad . \quad . \qquad (7)$$

Similar expressions apply to X and N.

The condition of stability is as before $Y_v N_r > N_v\,(Y_r - u_0)$. A more exact form of this condition in curvilinear motion is

$$\Delta = \begin{vmatrix} X_u, & X_v + r_0, & X_r + v_0 \\ Y_u - r_0, & Y_v, & Y_r - u_0 \\ N_u, & N_v, & N_r \end{vmatrix} < 0 \quad . \quad . \quad . \qquad (8)$$

or

$$X_u\Delta_1 + (X_v + r_0)\Delta_2 + (X_r + r_0)\Delta_3 < 0 \quad . \quad . \quad . \qquad (8a)$$

Let us now compare the stability of various airships and examine the variation of the stability criterion as the airship is yawed.

11.4. First of all it is proposed to consider the nature of the variation with β of the derivatives which are of the greatest importance in applying the above criteria to actual airships.

X_u.—From equation (6) it will be seen that when $\beta = 0$

$$X_u = \frac{2}{V}X_\beta$$

where $-mX_\beta$ is the drag of the airship. We have seen that the maximum value of β likely to occur in practice never exceeds 7 or 8 degrees in steady motion; we need, therefore, only consider values of β within a range of, say, $-8° < \beta < +8°$.

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* $(X_v + r_0)\begin{vmatrix} Y_u - r_0 & Y_r - u_0 \\ N_u & N_r \end{vmatrix}$ small compared to $X_u \begin{vmatrix} Y_v & Y_r - u_0 \\ N_v & N_r \end{vmatrix}$

† See Appendix III.*c*, p. 9. If the three important acceleration derivatives be retained, this equation takes the form

$$\left\{\lambda(1 - X_{\dot{u}}) - X_u\right\}\left[\lambda^2(1 - Y_{\dot{v}})(1 - N_{\dot{r}}) - \lambda\left\{Y_v(1 - N_{\dot{r}}) + N_r(1 - Y_{\dot{v}})\right\} + \begin{vmatrix} Y_v & Y_r - u_0 \\ N_v & N_r \end{vmatrix}\right] = 0$$

and the criterion $Y_v N_r > N_r(Y_r - u_0)$ is unaffected by inertia effects, since $Y_{\dot{v}}$ and $N_{\dot{r}}$ are both negative. Further, assuming $1 - X_{\dot{u}}$, $1 - Y_{\dot{v}}$ and $1 - N_{\dot{r}}$ to be approximately equal to 1.1, 2.0 and 2.0 respectively, the effect of accelerations appears to be such as to decrease values of the roots of equation (5) in the ratios 1:1.1, 1:2 and 1:2.

It is obvious that the drag of an airship in equilibrium will be the same at positive and negative values of β, hence $\partial X_\beta/\partial\beta = 0$ when $\beta = 0$, and experiments show this quantity to be small at all values of β within the defined range. X_β is negative and generally increases numerically with β, so that X_u is negative and increases numerically with β.

Y_u and N_u.—Both these derivatives are zero when $\beta = 0$, since an increase in the forward speed cannot affect Y_β and N_β which are zero at $\beta = 0$.

They are positive when β is not equal to 0, and Y_u is comparatively a small fraction of r_0, in conjunction with which it appears in (8) above.

X_v.—$X_v = 0$ when $\beta = 0$ and is positive when β is not equal to 0.

Y_v.—This derivative is important in its bearing on stability.

It is always negative and increases numerically with β; both terms within the brackets in the expression (7) increase with β (see Figs. 5 and 7). This applies to the parts of Y_β due to hull and fins (which are additive) and it is desirable to make Y_v as large as possible numerically, in order that the stability criterion may be satisfied. Any increase in the size of fins or angle of yaw will obviously affect this.

N_v.—Also an important derivative, is negative. The part of N_v due to fins is to be subtracted from the hull component, hence it is advisable to consider them separately.

$\partial N_\beta/\partial\beta$ due to hull decreases as β increases, and though N_β increases with β, N_v due to hull decreases numerically as β increases, since the predominant term in the expression for N_v is $\cos\beta\,\partial N_\beta/\partial\beta$.

On the other hand, $\partial N_\beta/\partial\beta$ due to fins increases numerically with β and so does $\sin\beta\,.\,N_\beta$, hence N_v due to fins increases numerically with β, but even though it never (in any existing design) approaches numerical equality with the hull component, the bearing of the fin dimensions and angle of yaw on the value of N_v is obvious; an increase in fin area or angle of yaw will decrease the numerical value of N_v—a desirable result if the stability criterion is to be satisfied.

We have already dealt with X_r Y_r and N_r in Section VII.; Y_r is positive, N_r is negative, and both are to a great extent due to fins; the greater the numerical values of both, the easier it is to satisfy the stability condition. Similar considerations apply to the pitching derivatives.

L_p.—If the axis of x coincide with the axis of the ship, then L_p due to hull is zero, and AL_p due to fins $= Ca^2N_r\lambda^2$ due to fins, where a is the distance of the C.P. of the forces on the fins from the axis of the airship.

If the axis of x be below the axis of the ship, then

$$L_p = -hL_v \text{ and } AL_v = -mhY_v$$

$$\text{or } L_p = \frac{m}{A}\,.\,h^2Y_v$$

where h is the distance between the axes.

Table 5, Appendix I., shows the full-scale values of the most important lateral derivatives for a number of airships in rectilinear motion.

It is evident from the figures that the hull contributes to the value of Y_v in not a small measure, particularly in the cases of models C, D and E, whereas the fin contribution to the values of Y_r and N_r is much greater than that of the hull. In all cases $Y_vN_r/N_v\,(Y_r - u_0)$ due to hull is but a small fraction of the total. With the models C, D and E, with a tail form of special design, the hull contribution is a greater percentage of the total, and indicates lines along which airship forms may be developed for greater stability with less fin area.

(It should be borne in mind that models C, D and E were not fitted with cars which would increase Y_r and hence help to satisfy the condition of stability.)

With regard to the variation of the values of the derivatives and stability criterion (8) with angle of yaw, reference should be made to Appendix III.*c*, in which will also be seen the effects of including acceleration derivatives of certain assumed values in the calculations of damping coefficients. The criterion itself, however, is independent of acceleration effects.

The following table applying to R.29*b* is reproduced from Table 1, Appendix III.*c*:—

Angle of Rudder (degrees).	0	5	10	15
Yaw (degrees).	0	3.6	5.1	6.3
$\frac{r_o}{u_o} \times 10^4$	0	2.93	4.08	4.98
$\frac{v_o}{u_o}$	0	−0.063	−0.086	−0.109
$\frac{X_u}{u_o} \times 10^4$	−5.08	−5.12	−5.15	−5.24
$\frac{X_v + r_o}{u_o} \times 10^4$	0	2.18	3.18	4.01
$\frac{Y_u - r_o}{u_o} \times 10^4$	0	−3.52	−4.71	−5.25
$\frac{Y_v}{u_o} \times 10^4$	−21.1	−33.7	−35.8	−40.7
$\frac{Y_r}{u_o} - 1$	−0.47	−0.47	−0.47	−0.47
$\frac{N_u}{u_o} \times 10^4$	0	0.028	0.044	0.061
$\frac{N_v}{u_o} \times 10^4$	−0.336	−0.261	−0.240	−0.192
$\frac{N_r}{u_o} \times 10^4$	−71.3	−72.8	−75.2	−77.4
$-\frac{\Delta}{u_o^3} \times 10^9$	−0.38	7.11	9.8	14.6
$-\frac{X_u \Delta_1}{u_o^3} \times 10^9$	—	—	8.03	—

In this case $(X_v + r_o)\Delta_2 + (X_r + v_o)\Delta_3$ is much less than, and it is always of the same sign as $X_u\Delta_1$, so that if the criterion $\Delta_1 > 0$ is satisfied, $\Delta < 0$ will be satisfied and it would appear that in all cases $\Delta_1 > 0$ is a sufficient condition for stability.

XII. *Motion Under Given Disturbing Forces*

12.0. The equations of motion, from which the conditions of stability of § 11 were deduced, may be used to determine the motion of an airship when external disturbing forces, such as gusts, are brought into consideration. It is, of course, impossible to solve the equations analytically unless the disturbing forces are expressible in certain mathematical terms, and even then the derivatives must be regarded as constant throughout the motion. One example of the application of the equations to such problems which may prove to be of considerable interest in the case of the airship is that of motion under certain rudder movements. Such an example was worked out by the author in connection with the investigation into the R.38 disaster. In one case considered the airship was assumed to be moving in a straight line and then the rudders moved rapidly hard over to one side, and then suddenly reversed to the extreme position on the other side. In the other case, with the ship turning steadily in a circle with the rudders hard over, the motion consequent upon a rapid motion of the rudders to the other extreme was calculated. It was hoped that a solution of these problems would be of assistance in determining the maximum stresses due to aerodynamic forces which an airship may encounter. In such problems as these, where the rudder movement cannot be conveniently expressed in simple mathematical terms, and where the resistance derivatives are not constant throughout the motion,* the equations must be solved by numerical methods.

A problem which is, however, of more general interest is that of the motion of the airship when the rudders are moved in simple harmonic motion, or conversely, the motion of the rudders consistent with a simple harmonic motion of the airship. In the latter form the numerical solution of the equations is simple, even if the range of variation of the angle of yaw extend beyond that in which the resistance derivatives are constant.

It must be remembered that the disturbed motion is accelerated and therefore the introduction of inertia coefficients is essential for determining the motion. Lack of reliable data concerning these coefficients renders an accurate solution impossible.

Motions of this type are, however, easily reproduced on full scale, and herein lies the reason for the importance of the problem. We have seen that a prediction of the equilibrium positions of an airship in circling flight from model results agrees with observations taken in flight, and that therefore it is probable that errors arising from ignoring scale effect are not appreciable. Consequently, it appears justifiable to state that the results of calculations of motions of the type considered (using model results only, with assumed values of the inertia coefficients) when compared with the observations taken of corresponding† motions on full scale, would prove invaluable and give definite data from which to determine the inertia coefficient or acceleration derivatives.

12.1. A detailed description of the analytical methods used is given in Appendix IV., together with details of calculations of the motion of R.32 to which the analysis was applied after having assumed certain values for acceleration derivatives.

The following summary of conclusions may be inserted here for completeness.

Assuming the rudders to move in simple harmonic motion amplitude $a_0 = 10°$, the direct solution of the differential equations is only applicable if $VT < 2{,}000$ feet. V is the velocity of the airship in ft./sec. and T the period of the rudder motion. Beyond this value of VT the coefficients in the differential equations (*i.e.*, functions of resistance derivatives) are not constant and the equations are not easily soluble.

* The angle of yaw becomes so great that the derivatives vary.

† Corresponding motions are defined as those produced by similar movements of the rudders.

If α_0 be 5° (or 20°) then $VT < 5{,}000$ (or $< 1{,}200$) feet defines the range of applicability, beyond which the angle of yaw during the motion exceeds about 1.15 degrees and the derivatives begin to change. Assuming V to be 100 feet per second, then for the values given above direct solutions of the equations will be possible only if $T < 50$ (or 12) seconds.

It was also found that the ratio v_{max}/r_{max} is approximately constant, where v_{max} and r_{max} are the maximum values of side-slipping and angular velocities of the ship during the oscillation. This constant is not the same constant as v/r in steady rotational motion (§8.5), but this may be due to the assumed values of inertia coefficients being in error. Further, v_{max} and hence also r_{max} vary directly as the amplitude of the rudder motion.

For cases outside the range defined the converse method is always applicable (see § 12.0). Quoting from the summary to Appendix IV.: "It is shown (taking $V = 100$ ft./sec.) that the total yawing moment on the airship is much greater when T is 5 secs. than when $T = 50$ secs.; the lateral force, however, is less, due to the small value of β_{max}. If the airship be flying steadily, either in a straight line or in a circle, and the rudders be put over to 20° in 5 secs., the maximum yawing moment acting is of the order of 350-400 tons-ft. when $V = 100$. The time taken for the ship to take up the equilibrium position consistent with the final rudder positions is about half a minute. It is pointed out that, if acceleration effects be included, the value of β and r given in the report may be approximately twice the actual values; the forces and moments, however, remain practically unchanged."

XIII. *Controllability*

13.0. An adequate discussion of controllability as such is a matter of some difficulty. The whole question appears to be so closely related to stability that it can hardly be considered independently. It is possible that the difficulty arises out of a certain vagueness of definition of the degree of controllability desired in any particular airship. It is probably impossible to define a satisfactory measure of controllability, since the behaviour of the airship will to a great extent depend upon the skill with which the control surfaces are operated, and the human element cannot be eliminated.

Two main conditions of flight present requirements which may be considered diametrically opposed to each other. On the one hand we may have to discuss the ease with which an airship may be flown on any path without unduly operating the rudders or, on the other hand, the readiness with which she will respond to the control surfaces while manœuvring.

If we consider the first point of view, it is evident that controllability is synonymous with stability, and a vessel for which the righting moment is appreciable when she is disturbed from a state of stable equilibrium will demand large rapid movements of efficient control surfaces in order that she may be manœuvred. Thus, if an airship be controllable according to the first requirement, she may be difficult to control in manœuvres.

From the second point of view, if the airship responds readily to any disturbance, it is evident that the righting couple cannot be great. It may be negative in rectilinear flight and the airship would then be unstable and require constant attention to the rudders to enable her to fly along a straight path. Hence, an airship easy to manœuvre might be difficult to fly straight.

13.1. The following simple criterion has been suggested for comparing the controllability of airships in rectilinear motion*:—

* R. & M. 802.

"The ease with which an airship may be controlled in straight flight may be measured by the value of the ratio of the moment due to turning the rudder through a definite angle, to the rate of change of yawing moment with angle of yaw when the rudder is amidships. The greater the value of this ratio the greater the control."

That the criterion is open to serious objection as a measure of the efficiency of control or degree of controllability is beyond doubt. Imagine the fixed fin of an airship increased in size, the control surfaces remaining the same, then the ratio increases since the yawing moment decreases. The ship becomes more stable and thus less ready to respond to the rudders. On the other hand, if the area of control surfaces be increased, the fixed fins remaining the same, the ratio will increase since the numerator becomes greater and the denominator less.

If, however, the total stabilising area (*i.e.*, fixed fins+rudders) remain unaltered, but the rudder area increased at the expense of the fixed fins, then the denominator remains unchanged, whereas the numerator becomes greater.

Table 6, Appendix I., shows the value of the ratio in the case of some models for which data are available.

13.2. It would appear that a more satisfactory measure of controllability, defined as the readiness with which an airship will respond to her rudders, is given by the rate of change of angular velocity with angle of rudder, $\partial r/\partial \alpha$. Since the angular velocity at any given speed is inversely proportional to the radius of curvature of the path, Figs. 9-9b give an indication of its rate of change with rudder inclination. It is obvious that $\partial r/\partial \alpha$ increases as the fin area decreases and becomes infinite at $\beta=0$ when $N_r/N_v=K\lambda$. From considerations similar to those applied in §8.10, it becomes negative at $\beta=0$ when $N_r/N_v<K\lambda$, *i.e.*, when the airship is unstable in rectilinear motion. It is easy to show that in curvilinear stable motion, if the rudders are moved, the initial value of $\partial r/\partial \alpha$ is approximately* inversely proportional to $N_r-K\lambda N_v$, and thus the degree of controllability is inversely proportional to the degree of stability.

13.3. The effect of altering the rudder area, on the diameter of the turning circle in curvilinear flight, is also difficult to consider in general terms. Calculation can with ease be conducted on any particular ship, given the necessary data. One interesting conclusion derived from such calculations applied to R.32 is the following:—For a certain range of rudder areas (including the area of the R.32 rudders) a variation of 20 per cent. in the area will not appreciably affect the magnitude of the diameter of turning circle; hence when the airship is turning with the rudders inclined, increasing the rudder areas will not affect the curvature of the path.

This result may appear at first sight unbelievable, but reference to Figs. 9-9b will indicate the manner in which it becomes possible. For imagine the area of fixed fin to remain unchanged while the rudder area is increased, the ordinates of the N_v/VN_r curve with $\alpha=0$, or rudders amidships, will be decreased by an amount proportional to the increase in the area of the rudders. The difference between the ordinates of the N_v/VN_r curves for $\alpha=0$ and $\alpha=10^\circ$ say, will, however, be increased by the same amount, and thus the point of intersection of the curve $\alpha=10^\circ$ with the curve $R\beta$ constant may be unchanged. The obvious conclusion which follows from the result is that, after a state of steady motion has been attained, there is no transverse force on the rudder.

13.4. Mention might be made of a certain anomalous condition which sometimes occurs in symmetrical flight, *i.e.*, with the plane of symmetry vertical. If the aerodynamic disturbing moment be greater than the static righting couple it may become necessary to raise the elevators when flying nose down. Should the elevators be raised to the maximum in order to maintain equilibrium at a

* Assume N_v and N_r to have the values at $\beta=0$.

certain speed, an increase in speed will increase the aerodynamic couple while leaving the static couple unaffected and increase the downward pitch, which will be accompanied by loss of height. This phenomenon of "reversal of control" is closely allied with the problem of longitudinal equilibrium referred to in § 8.0, but bears no relation to the reversed control which was discussed in § 8.10, and which is a purely aerodynamical effect independent of the speed and distribution of weight of the airship.

XIV. *Conclusion*

14.0. A few brief remarks may be added, in conclusion, regarding the application of the principles and data presented in the present paper to the design of an airship from a purely aerodynamical point of view. It will, of course, be borne in mind that figures applicable to a certain airship may not strictly hold good for another; it is proposed merely to indicate lines along which a designer may proceed when considering the shape of the envelope and dimensions of the fixed fins and control surfaces, in order that an airship may have a specified performance. A model made to the preliminary design, based on these data, may then be tested in a wind channel if desired.

14.1. *Speed.*—The dimensions of any proposed airship will probably be determined from considerations of the shed accommodation and the work which it is intended the airship should perform, *i.e.*, the load to be carried and the speed, both of which depend more or less upon the radius of action. Having decided upon these requirements, the general results given in §§ 4.0-4.9 will serve as a guide in choosing the form of hull likely to give the minimum resistance, and a knowledge of the resistance coefficient of the complete airship estimated from the tables of § 4.9 will indicate the power necessary to propel the airship at a certain speed. It is seen from the tables in § 4.9 that only about 3 per cent. can be saved on resistance by adopting fins of V section to enclose the fin bracing wires. This is negligible when compared with the decrease in fin and control efficiency associated with such fins (see Tables 3 and 4, Appendix I., R.38, fins 1 and 2). Larger fins and control surfaces would be required to give the same degree of stability and controllability as flat fins, and this would result in increased drag and weight. It has been mentioned (§ 4.4) that adopting a tail of elliptical cross section tapering to a fine edge at the extreme after end does not appreciably alter the resistance coefficient. Less vertical fin area would in this case be needed, but the saving in fin resistance would probably not amount to more than 2 per cent., though much would be gained in strength and increased tail lift with a decrease in the weight of the fins. It would thus appear safe to predict that no great improvement in *resistance* is likely to follow from modifications along these lines, whereas the adoption of a hull form embodying but very little cylindrical body, and with a continuously curved head would probably result in a considerable saving of power.

14.2. *Stability.*—Since the conditions of stability in longitudinal and lateral motions are exactly similar, one only need be considered. It is evident that to obtain a great degree of stability Y_v, Y_r and N_r must be as large as possible numerically, and N_v as small as possible.

This involves adopting a hull form for which the lateral force coefficient is large and the moment coefficient small. These quantities have been seen to be roughly of the same magnitude on various models and thus indicate that one body has not much superiority over another unless the ratio of elevation area to volume is greater. It is fairly obvious, however, that the adoption of a long length of cylindrical body is, as in the case of resistance, a decided disadvantage. A judicious choice of fins is of the greatest importance in this connection. They should be as wide as possible in comparison with their length and situated as far aft as is consistent with safety; then the moment due to fins (which has to be

subtracted from the moment on the hull) is increased and a low value for N_r follows, accompanied by an increase in the value of N_v, since N_r due to fins is proportional to λ^2.

It is a simple matter to calculate approximately the stability criterion in the case of any design from the data given, provided of course that the form of hull and disposition of fins do not involve any serious fundamental innovations. Even then a good estimate of the degree of stability and attitudes of equilibrium can be made. It is obvious that the difficulty arises with regard to Y_r. If Y_r due to hull is likely to be appreciably different from the value assumed in Section VIII., erroneous deductions might result. The method used when dealing with models C, D and E might be mentioned as indicating the lines along which to proceed. It was felt in this case that the modified form of tail was appreciably different from normal design and was likely to give rise to a larger value for Y_r due to hull. Accordingly, a probable value was assumed and $R\beta$ calculated as in § 8.4. Using this, Y_r on the complete ship is derived and the difference between this value and Y_r due to the hull should be roughly equal to $CN_r/m\lambda$ due to fins. If this is not the case a trial and error method must be followed until it is found that Y_r due to the fins, as calculated, is equal to $CN_r/m\lambda$ due to fins as obtained experimentally. In the cases considered Y_r due to the hull was found to be 0.22 ν_0, or roughly between two and three times the value for a hull of normal design. $\bar{R}\beta$ was also found to be rather less than the mean value given in Section VIII.

An application of this method to the three models C, D and E, and a calculation of the area of stabilising surfaces required to ensure stability, resulted in the following conclusion:—If the head and tail of a model airship hull remain the same, but the length of cylindrical body be altered, the fin area required for neutral stability will be approximately proportional to the longitudinal elevation area of the model. The same result was derived for another body which, though not an airship model, was of a similar form.

14.3. *Equilibrium and Controllability.*—In determining the ratio of the area of control surfaces to that of fixed fin or to the total area, the rate of turn desired at any particular rudder setting must first of all be decided. The data given for the efficiency of control surfaces will then be of assistance in arriving at the required result. All that is necessary is to calculate the moment due to the control surfaces at a negative angle of rudder, and add the result to the yawing moment with rudders amidships at positive angles of yaw. Dividing by $V \cdot N_v$, plot the result against β and observe the intersection of the curve with the curve $R\beta = \text{const.}$ (§ 8.2). The position of equilibrium is then available and the rate of change of angular velocity with rudder inclination follows.

14.4. *Pressure Distribution.*—The results of pressure distribution experiments, typical curves relating to which are shown in Fig. 12, will prove invaluable to a designer in deciding upon the most satisfactory positions for valves and for carrying out calculations of the stresses in the structure. In this connection a consideration of the most violent oscillation likely to be encountered by an airship in motion as calculated according to the methods mentioned in Section XII., and described in greater detail in Appendix IV., should render it possible to arrive at a fairly accurate estimate of the maximum stresses to be borne by the structure, and thus to minimise the risk of such a disaster as met the ill-fated R.38 ever meeting another airship.

In conclusion, reference might be made to a paper by Professor Bairstow* in which the aerodynamic loading of airships is discussed, and which shows how the results of pressure distribution experiments may be applied to determine the shearing force and bending moments on an airship considered as a loaded beam.

* R. & M. 794.

This, however, is outside the scope of the present paper, which is merely intended as a survey of the aerodynamic data which may be applied to such problems and to a determination of the general motions of an airship.

List of the Reports and Memoranda of the Advisory Committee for Aeronautics and the Aeronautical Research Committee, to which reference is made. These reports are published by H.M. Stationery Office, with the exception of T.1320.

R. & M. No.	Title.	Authors.
244	Method of support for models.	Pannell and Campbell.
246	Experiments on model airships.	Pannell and Campbell. assisted by Pell and Powell.
257	The stability of airships. Mathematical theory and illustrations of its use.	Bairstow, Relf and Nayler.
311	The resistance of certain streamline bodies.	Pannell and Campbell.
361	Experiments on a model of the German rigid airship R.33.	Pannell and Jones.
457	Forces and moments upon a complete model airship, type S.S.Z.	Frazer and Simmons.
504	Experiments in a wind channel on elongated bodies of approximately streamline form. Part I.: Static pressure correction.	Pannell, Jones and Pell.
590	The dependence of the resistance of bodies upon acceleration as determined by chronograph analysis.	Frazer and Simmons.
593	Speed effect on the stability of R.80 airship model.	Jones and Pell.
600	The distribution of pressure over the surface of airship model U.721, with a comparison with the pressures over a spheroid.	Jones and Williams.
602	The stability of airships: general case. The mathematical theory and its application, with appendix.	Nayler and Jones.
607	Experiments in a wind channel on elongated bodies of approximately streamline form. Part II.: The effect of form on resistance.	Pannell & Jones.
612	On the effects of acceleration on the resistance of a body.	Cowley and Levy.
613	Measurement of the effect of accelerations on the longitudinal and lateral motion of an airship.	Relf and Jones.
619	The prediction from model experiments of the resistance of an airship of the 23 class.	Pannell, Jones and Pell.
623	The inertia coefficients of an ellipsoid moving in a fluid.	Lamb.
653	Balanced controls for rigid airships.	Jones and Williams.
665	A method for determining the rotary derivatives M_q and N_r.	Simmons and Bateman.
714	Experiments on a model of rigid airship R.29.	Jones, Williams and Bell.
716	The application of the results of model airship experiments to full-scale turning.	Jones.

R. & M. No.	Title.	Authors.
748	Acceleration derivatives in the case of a body moving in ideal fluid.	Jones.
749	The equilibrium of airships in curvilinear flight; an addendum to R. & M. 716.	Jones.
751	The stability of airships.	Jones and Williams.
779	Experiments on a model of rigid airship R.32, together with a comparison with the results of full-scale turning trials, and a consideration of the stability of the ship.	Jones, Williams and Bell.
780	Aerodynamic pressure on an airship hull in curvilinear flight.	Jones.
781	The motion of airships under certain imposed movements of the rudders.	Jones.
794	The aerodynamic loading of airships.	Bairstow.
799	Experiments on a model of rigid airship R.38.	Jones and Williams.
801	The pressure distribution over a model of the hull of airship R.33.	Jones and Bell.
802	Experiments on models of a rigid airship of new design.	Jones, Williams and Bell.
808	Pressure plotting on fin and rudder of a model of R.32.	Williams and Bell.
815	Measurements of normal force and pitching moment on rigid airship R.33.	Frazer and Bateman.
827	The prediction of the resistance of the rigid airship R.33.	Frazer and Gadd.
858	The resistance of a spheroid with special reference to the effect of wires placed around the model in a plane at right angles to the axis.	Jones and Bell.
T.1320	The stability of airships based on the results of Italian experiments (not published).	Jones and Williams.

Appendix I.

Summary of experimental results on Models of recent British airships tested at the National Physical Laboratory, and drawings showing the models.

Table 1 Lateral and normal force coefficients on the hull. §6.2 (ii.).
,, 2. Yawing and pitching moment coefficients on the hull. §6.2 (iii.).
,, 3. Force coefficients on the fins. §6.4.
,, 4. Force and moment coefficients on the rudders. §6.5.
,, 5. Full-scale derivatives and stability criterion. §11.4.
,, 6. Values of controllability criterion. §12.1.

Fig. 1. Drawing of R.29, showing R.29a and R.29b.
,, 2. ,, ,, R.32.
,, 3. ,, ,, R.33.
,, 4. ,, ,, R.38.
,, 5. ,, ,, R.38 fins.
,, 6 ,, ,, Models C, D and E.

TABLE 1.

HULL ONLY: FORCE COEFFICIENTS.

In yaw: Lateral force $\div \rho V^2 A_e$.

In pitch: $-$ Normal force $\div \rho V^2 A_p$.

Where A_e and A_p represent elevation and plan area respectively.

Angle of Incidence (degs.).	Yaw.				Yaw and Pitch.			Pitch.
	C	D	E	R.38*	R.33	R.32	R.29	D
0	0	0	0	0	0	0	0	0
1	0.0013	0.0012	0.0009	0.0011	0.0008	0.0008	0.0007	0.0009
2	0.0027	0.0024	0.0020	0.0021	0.0011	0.0016	0.0016	0.0017
3	0.0042	0.0038	0.0031	0.0032	0.0020	0.0024	0.0026	0.0025
4	0.0058	0.0052	0.0044	0.0045	0.0026	0.0033	0.0036	0.0035
5	0.0072	0.0066	0.0059	—	—	0.0043	0.0047	0.0046
6	0.0091	0.0084	0.0074	0.0072	0.0042	0.0054	0.0059	0.0057
8	0.0134	0.0128	0.0118	0.0110	0.0059	0.0085	0.0085	0.0084
10	0.0190	0.0185	0.0169	0.0150	0.0089	0.0116	0.0122	0.0115
12	—	—	—	0.0220	0.0119		0.0162	—
15	0.0380	0.0362	0.0332	—	—	0.0218	—	0.0212
16	—	—	—	0.0379	0.0185	—	0.0263	—
20	0.0640	0.0606	0.0575	0.0594	0.0254	0.0354	0.0400	0.0346
Area A. (sq. ft.)	1.51	1.69	1.965	1.246	2.70	1.95	1.48	1.534
	Elevation				Elevation and Plan			Plan

*R.38 had a small keel, see fig. 4.

TABLE 2.

HULL ONLY: MOMENT COEFFICIENTS.

In yaw: Yawing moment $\div \rho V^2$ volume.

In pitch: Pitching moment $\div \rho V^2$ volume.

Angle of Incidence (degs.).	Yaw.				Yaw and Pitch.			Pitch.
	C	D	E	R.38	R.33	R.32	R.29	D
0	0	0	0	0	0	0	0	0
1	0.012	0.013	0.014	0.012	0.014	0.014	0.014	0.013
2	0.025	0.025	0.027	0.026	0.029	0.027	0.027	0.026
3	0.036	0.037	0.041	0.040	0.042	0.042	0.040	0.040
4	0.048	0.050	0.054	0.054	0.055	0.055	0.053	0.053
5	0.059	0.062	0.065	—	—	0.069	0.066	0.066
6	0.070	0.072	0.076	0.079	0.082	0.081	0.079	0.079
8	0.088	0.090	0.093	0.099	0.106	0.105	0.103	0.103
10	0.102	0.103	0.107	0.120	0.131	0.127	0.146	0.125
12	—	—	—	0.135	0.151	—	—	—
15	0.120	0.125	0.132	—	—	0.178	0.184	0.175
16	—	—	—	0.559	0.187	—	—	—
20	0.122	0.128	0.137	0.181	0.223	0.218	0.217	0.221
Vol. (cu. ft.)	0.520	0.599	0.715	0.370	1.22	0.680	0.444	0.599
$(\text{Vol.})^{2/3} = l^2$ (sq. ft.)	0.646	0.711	0.800	0.515	1.14	0.774	0.587	0.711
A/l^2	2.34	2.38	2.46	2.42	2.45	2.53	2.51	2.16

TABLE 3.

FIN EFFICIENCY.

In yaw: (Vertical fins). Lateral force due to fins
In pitch: (Horizontal fins). − Normal force due to fins
$\Big\} \div \rho V^2 A_f$.

where A_f is the area of stabilising and control surfaces (the latter amidships).

Angle of Incidence (degs.).	Vertical Fins.								Horizontal Fins.	
				R.38						
	C	D	E	1	2	3	R.32	R.29	D	R.33
0	0	0	0	0	0	0	0	0	0	0
1	0.016	0.012	0.012	0.007	0.008	0.008	0.011	0.004	0.007	0.013
2	0.034	0.025	0.030	0.012	0.016	0.020	0.022	0.012	0.020	0.032
3	0.053	0.045	0.049	0.024	0.033	0.036	0.037	0.021	0.032	0.050
4	0.071	0.071	0.074	0.034	0.045	0.050	0.053	0.038	0.046	0.071
5	0.100	0.100	0.098	—	—	—	0.073	0. —	0.065	—
6	0.132	0.132	0.137	0.058	0.087	0.092	0.098	0.087	0.081	0.127
8	0.185	0.190	0.185	0.106	0.127	0.139	0.153	0.148	0.125	0.182
10	0.240	0.245	0.245	0.153	0.194	0.200	0.226	0.227	0.178	0.258
12	—	—	—	0.198	0.240	0.251	—	0.306	—	0.333
15	0.356	0.356	0.375	—	—	—	0.409	—	0.316	—
16	—	—	—	0.290	0.343	0.375	—	0.462	—	0.506
20	0.480	0.480	0.444	0.404	0.460	0.507	0.589	0.636	0.462	0.746
A. Fin area (sq. ft.)	0.0429	0.0429	0.0429	0.0639 V section.	0.0639	0.0666	0.100	0.093	0.0963 V section.	0.175

TABLE 4.

EFFICIENCY OF CONTROL SURFACES AT ZERO YAW.

Model.	Angle of Control (degs.).	Force $\div \rho V^2 A_c$.	− Moment $\div \rho V^2 \lambda_2 A_c$.	λ_2 (ft.).	A_c (sq. ft.).
C. in Yaw	10	0.248	0.232	1.835	
D in Yaw	10	0.243	0.224	2.005	0.023
	20	0.456	0.425		
E in Yaw	10	0.219	0.201	2.26	
R.33 in Yaw	10	—	0.186	2.725	0.039
	20	—	0.335		
R.29 in Yaw	10	0.182	0.169		
R.29a in Yaw	10	0.177	0.169	1.99	0.024
R.29b in Yaw	10	0.153	0.140		
	10	—	0.232		
R.38 fins 1 in Yaw ...	20	—	0.478	1.75	0.0155
	30	—	0.620		
R.38 fins 2 in Yaw ...	10	—	0.243		
	20	—	0.501	1.75	0.0155
	30	—	0.646		
R38. fins 3 in Yaw ...	10	—	0.248	1.75	0.0161
	20	—	0.496		
	10	0.185	0.193		
R.32 in Yaw	20	0.372	0.372	2.23	0.0258
	30	—	0.417		
D in Pitch	10	0.356	0.335	1.995	0.025
	20	0.651	0.612		

A_c = Area of Control Surfaces (sq. ft.).
λ_2 = Distance from C.B. to rudder hinge (ft.).

TABLE 5.

RESISTANCE DERIVATIVES FULL-SCALE AT ZERO INCIDENCE.

Dimensions.	L^{-1}	L^3	L^4	Non-Dimensional.	
	$-\frac{Y_v}{u_0} \times 10^3$	$-\frac{CN_v}{u_0} \times 10^{-3}$	$-\frac{CN_r}{u_0} \times 10^{-6}$	$1-\frac{Y_r}{u_0}$	$-\frac{Y_v N_r}{N_v (u_0 - Y_r)}$
Model C Hull only ...	1.15	6.81	0.58	0.78	0.125
Yaw with fins	1.45	5.73	1.41	0.52	0.60
D Hull only	1.06	8.22	0.88	0.78	0.145
Yaw with fins	1.26	7.01	1.82	0.55	0.60
E Hull only, no cars ...	0.81	10.58	1.12	0.78	0.13
Yaw with fins	0.90	9.30	2.24	0.58	0.38
D Hull only	0.63	8.57	0.16	0.9	0.013
Pitch with fins	1.01	7.63	1.82	0.4	0.61
R.29 Yaw. Hull only ...	0.98	2.00	0.113	0.9	0.064
R.29 Yaw, Hull only and cars	1.84	2.01	0.113	0.9	0.115
R.29 Yaw with fins and cars	2.29	1.75	0.484	0.26	2.4
R.29a Yaw with fins and cars	2.21	1.80	0.477	0.41	1.43
R.29b Yaw with fins and cars	2.21	1.83	0.393	0.47	0.96
R.33 Yaw, Hull only ...	0.70	3.98	0.20	0.9	0.062
R.33 Yaw with fins and cars	1.35	3.23	0.895	0.44	0.85
R.32 Yaw, Hull only ...	1.17	3.04	0.083	0.9	0.036
R.32 Yaw, Hull only and cars	1.47	3.04	0.083	0.9	0.044
R.32 Yaw with fins and cars	1.96	2.44	0.56	0.36	1.25
R.38 Yaw, Hull only ...	1.025	5.30	—	0.9	—
R.38 Yaw, Hull and cars	1.23	5.36	—	0.9	—
with fins 1 and cars	1.49	4.46	0.91	0.52	0.60
Ditto, fins 2	1.58	4.41	0.99	0.43	0.83
Ditto, fins 3	1.64	4.21	1.01	0.41	0.96

TABLE 6.

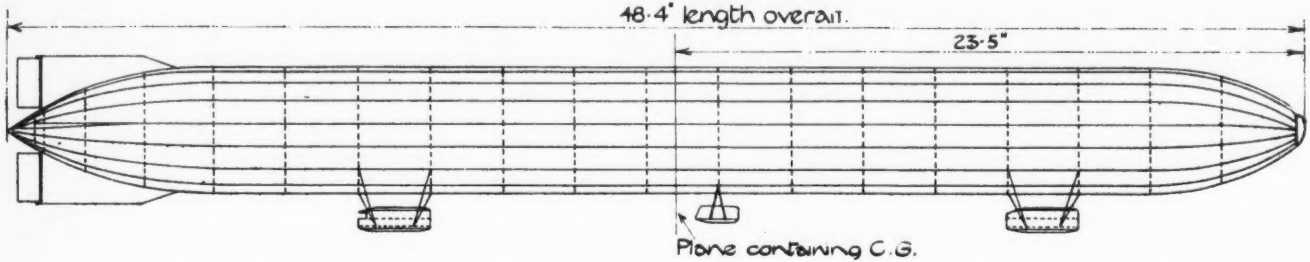
CRITERION FOR CONTROLLABILITY.

	$\frac{\partial CN_\beta}{\partial \beta} \div \rho V^2 vol.$	CN_β due to rudder $\div \rho V^2 vol.$	CN_β due to rudder $\partial CN_\beta / \partial \beta$ (3).
R.29	0.0122	0.0182	1.50
R.29a	0.0125	0.0182	1.46
R.29b	0.0127	0.0151	1.19
R.32	0.0111	0.0163	1.46
R.33	0.0108	0.0143	1.32
R.38, fins 1	0.0111	0.0170	1.53
R.38, fins 2	0.0110	0.0178	1.63
R.38, fins 3	0.0105_5	0.0182	1.73
Model C	0.0101_5	0.0187_5	1.85
Model D	0.0106_8	0.0172	1.62
Model E	0.0120	0.0146_8	1.22

Mean of (3) for R.29, R.29a, R.32 and R.33 = 1.43.*

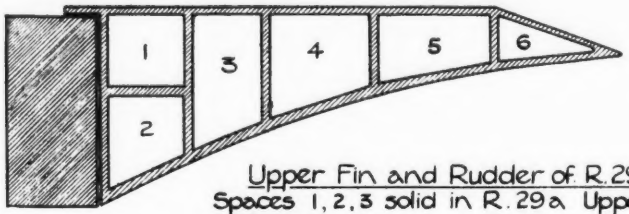
* R.29b has been omitted since all the fabric on the upper fin had been removed.

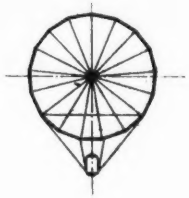
MODEL OF R.29.



SCALE OF MODEL 1/133·8 Full Size

SCALE OF DRAWING 1 0 1 2 3 4 5 6 7 8 9 inches



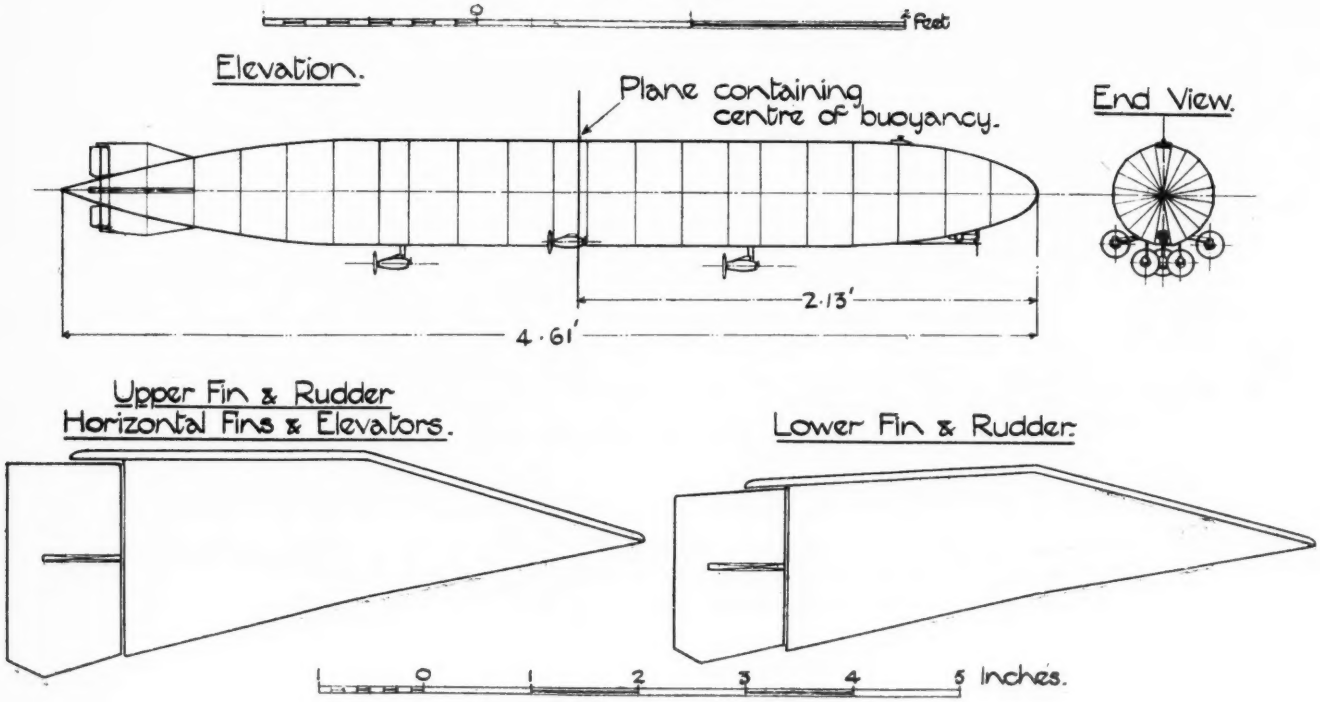


Upper Fin and Rudder of R.29b.
Spaces 1, 2, 3 solid in R.29a Upper Fin.
All Spaces, 1-6, " " R.29 " "
Horizontal Fins the same as R.29 Upper Fin.
Lower Fin slightly larger, the Fin being fixed to the hull along a flat, the other three being on ridges.

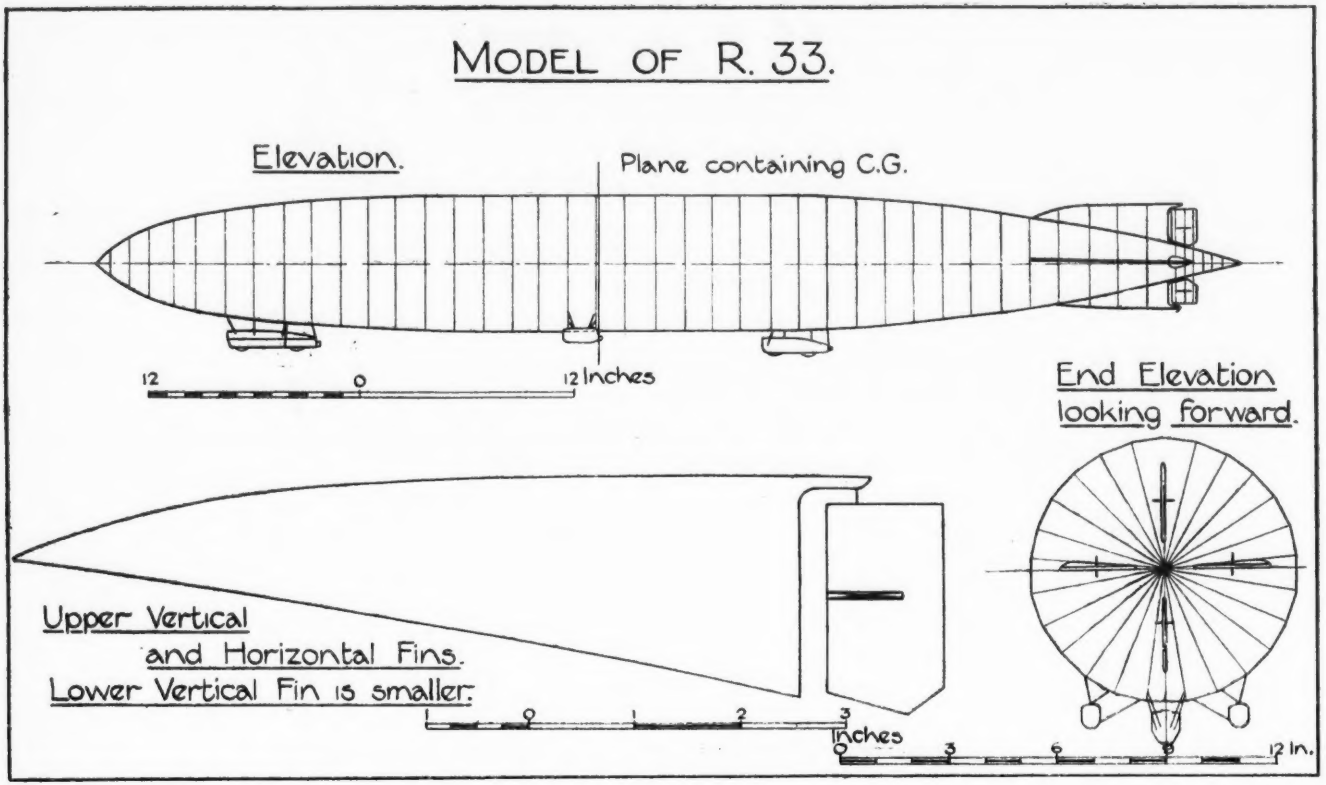
SCALE OF FIN :- 0 1 2 3 inches.

APPENDIX I., FIG. 1.

MODEL OF R.32.

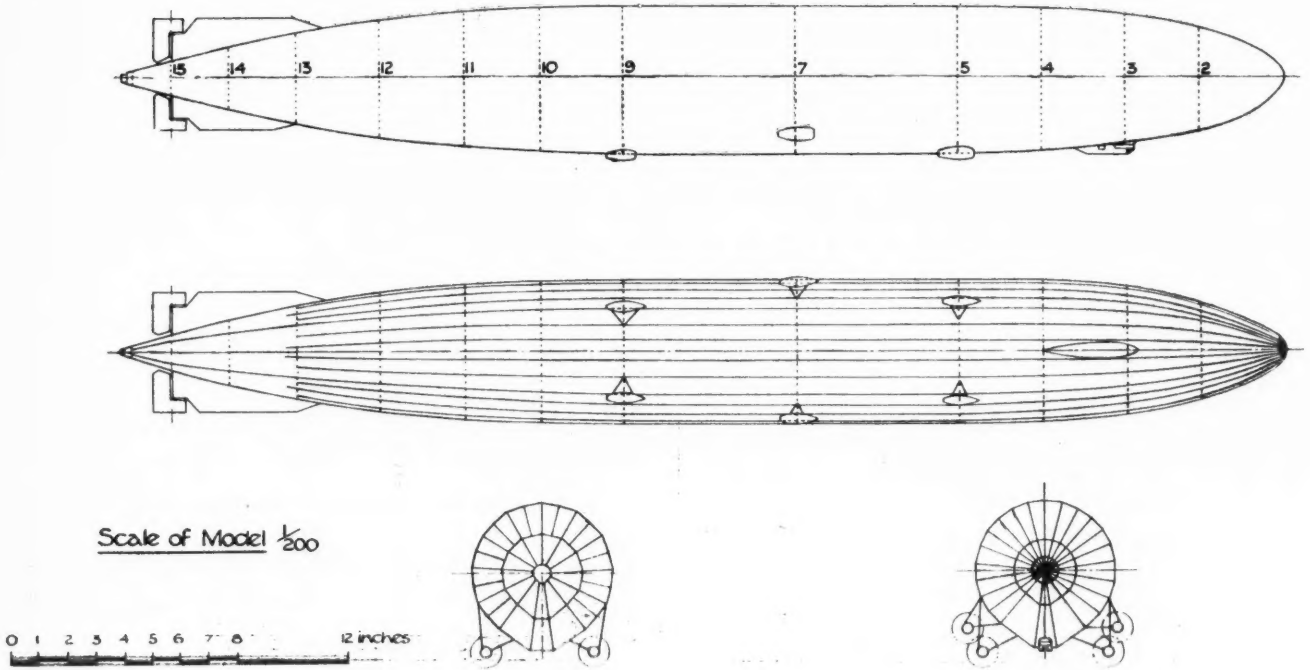


APPENDIX I., FIG. 2.



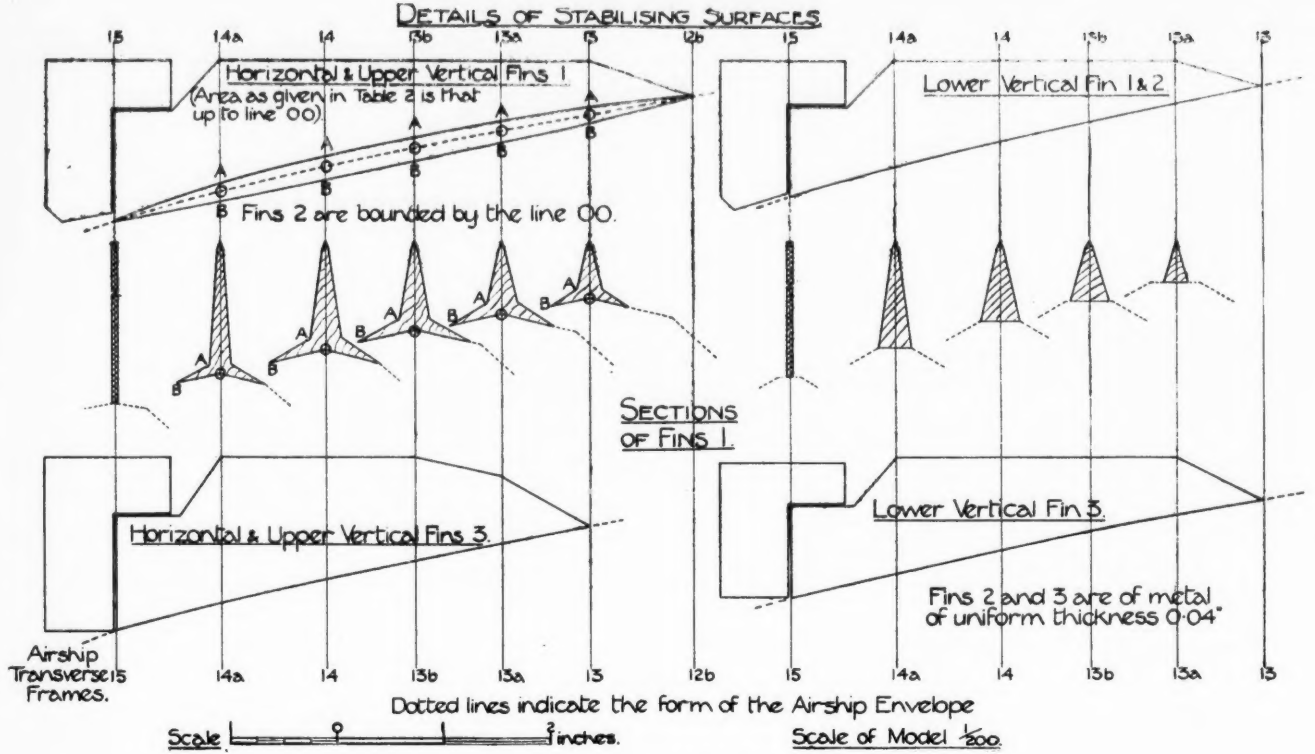
APPENDIX I., FIG. 3.

MODEL OF R.38.



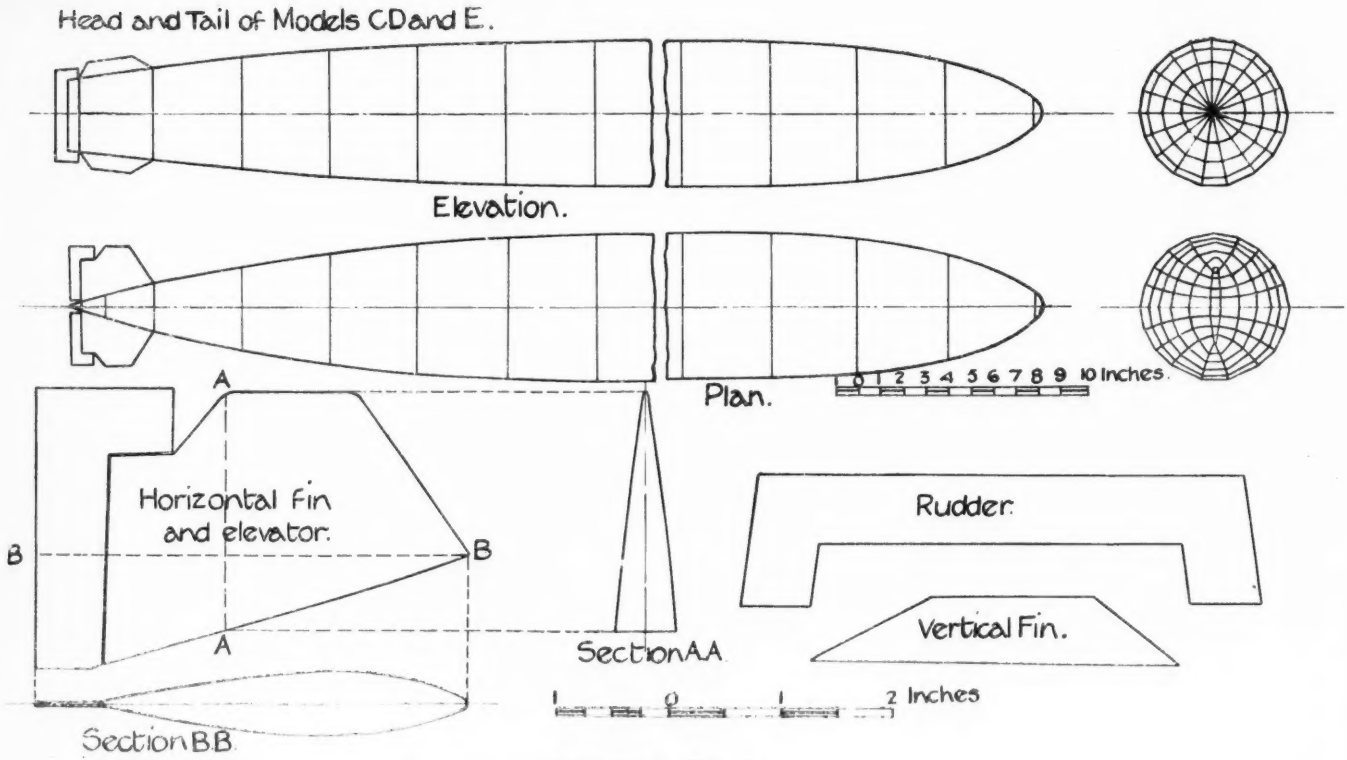
APPENDIX I., FIG. 4.

FINS OF R.38 MODEL.


DETAILS OF STABILISING SURFACES
Horizontal & Upper Vertical Fins 1
(Area as given in Table 2 is that up to line 00)
Fins 2 are bounded by the line 00.
Lower Vertical Fin 1 & 2
SECTIONS OF FINS 1.
Horizontal & Upper Vertical Fins 3.
Lower Vertical Fin 3.
Fins 2 and 3 are of metal of uniform thickness 0·04"
Airship Transverse Frames.
15 14a 14 13b 13a 13 12b
Dotted lines indicate the form of the Airship Envelope
Scale 0 1 2 inches.
Scale of Model 1/200

APPENDIX I., FIG. 5.

Appendix I., Fig. 5.



Appendix I., Fig. 6.

Appendix II.

R. & M., No. 780. "Aerodynamic Pressure on an Airship Hull in Curvilinear Flight." By R. Jones, M.A.

Appendix III.*a*

R. & M., No. 257. "The Stability of Airships—Mathematical Theory and Illustrations of its Use." By L. Bairstow, A.R.C.Sc., E. F. Relf, A.R.C.Sc., and J. L. Nayler, B.A.

Appendix III.*b*

R. & M., No. 602. "The Stability of Airships: General Case. The Mathematical Theory and its Application, with Appendix." By J. L. Nayler, B.A., and R. Jones, M.A.

Appendix III.*c*

R. & M., No. 751. "The Stability of Airships." By R. Jones, M.A., and D. H. Williams, B.Sc.

Appendix IV.

R. & M., No. 781. "The Motion of Airships Under Certain Imposed Movements of the Rudders." By R. Jones, M.A.